THE BOOK WAS DRENCHED

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PREFACE

THE following pages do not require in the reader any knowledge of Mathematics except such Geometry as has been acquired at school, and very little of that.

I am aware that to a number of thinkers of great powers the very sight of a diagram is odious. But the feeling of repulsion is really only similar to the dislike of a new language. To read German in old characters is disagreeable to many persons, and the difficulty of Russian and Hebrew partly depends on the alarming aspect of the alphabets of those languages. But the geometrical method is so charming when once it is grasped, and enables somewhat recondite laws to be so clearly seen, that its mastery is worth an effort.

The thanks of all English students are due to Professor Marshall, to Fleeming Jenkin, and Mr. Stanley Jevons, for introducing this method into England; and I shall not easily forget the pleasure which I felt, when about thirty years ago the first-named of these writers initiated me into the geometrical method of studying Economics which he has done so much to develop and improve.

The title which I have given to this work is perhaps open to the objection that the curves employed are rather graphic than truly geometrical. An examination of them will, however, show that they are more than mere 'graphs.' For by a graph I understand a curve

which merely aims at presenting a collection of facts to the eye, without any known law behind it; as, for instance, the diagram or graph representing the rise and fall of a barometer. When, however, a law can be discovered governing the form of the graph, it ceases to be a mere graph and becomes promoted to the dignity of a curve. Thus, for example, Maxwell in his theory of heat calls the isothermal and adiabatic lines, which are used to express changes of volume in a gas due to pressure and temperature, 'curves,' and he explains the laws which govern them.

The word 'diagrammatic' would perhaps have better expressed what was intended, but it would have been unusual and open to misconstruction. I therefore fell back on the use of the word 'geometrical'; for though I must admit there are objections to it, yet it seemed most nearly to express the view of the subject which I wished to present.

In Chapter XI I have endeavoured to explain the curves which ought, I think, for the future to be associated with the name of Professor Marshall.

It will be observed that I have treated his system as correlative to, and capable of being derived from and translated into, other systems.

From some correspondence I have had with him I ought, however, to state that (if I understand him rightly) he would not altogether accept this view. As will be seen by a reference to his work on Economics, he lays stress on the difference between short-period and long-period curves. His foreign exchange curves are intended to be used chiefly for illustration, and as

only applicable to long-period normal supply, and he is inclined I think to consider some of the curves which I have put forward as applicable only to short-period supply. If this is so it is at all events not my intention. All the curves mentioned in this book are intended to be applicable to states of equilibrium, reached after temporary oscillations have ceased; or rather, since all things are in a state of perpetual flux, as instantaneous photographs taken at times when the market conditions are normal. Thus, when I speak on p. 73 of surplus value, the successive costs shown by the curve Σ P are not meant to refer to succession in time, still less merely to short-period phenomena, but only to show that out of every batch of articles produced, as, for instance, a days' output of coal, part has been got more easily than other parts; and this may, and most likely will, continue through the whole life of the mine, and in all the mines in the country.

It does not seem to me, nor do I understand Professor Marshall to say (see *Principles of Economics*, Book v, ch. iv, p. 416, 1890 ed.), that there is any fundamental difference between short-period and long-period curves. Just as the sciences of Dynamics and Statics are really governed by the same underlying principles, so a short-period curve, which represents the condition of things during a disturbance, ultimately depends on the same principles as those which govern a long-period curve, which represents a condition of comparative rest. Therefore curves of the character described appear applicable both to short periods and long periods, provided only care is taken in their

construction to adapt them to the use for which they are designed.

I know how difficult it is to explain one's own views, not to speak of trying to explain those of another. I could not leave undescribed the pioneer work that Professor Marshall has performed; and therefore I can only say that if I have misapprehended the character of his curves, it has not been for want of a careful attempt to understand them, and to ascribe to them and to him the honour that they justly deserve.

A writer who attempts to use curves to explain Political Economy is open always to two dangers. If he makes the curves very simple, he is liable to be accused of not having seen the necessary qualifications to which economic laws are subject, and to be considered crude and wanting in subtlety. If, on the other hand, he attempts to enunciate every proposition with all the necessary safeguards, qualifications, exceptions, and explanations, his system becomes over-loaded, like the epicycles that were heaped upon epicycles in the closing period of the Ptolemaic Astronomy. I have however preferred simplicity even at the risk of being considered inadequate. It must surely be a very unreasonable person who expects that a mathematical method can be proposed of dealing with some of the most complicated problems that are perplexing the world, which shall on the one hand be so simple that those can understand it who are but little versed in Mathematics, and on the other so complete at all points that it can instantly resolve all difficulties, even of a recondite and exceptional character.

The curves I have given are true on the assumptions made regarding them. In proportion as they are used to aid in the comprehension of more difficult problems, so must they be added to, and qualified, just as a mechanical engineer is obliged successively to take account of inertia of parts, friction, heat-expansion, changes of texture of metal through use, liability to rust, metallic fatigue, and a number of other considerations, in designing a machine. But no one will blame a simple exposition of the laws of Mechanics or Dynamics because it treats metals as non-elastic, or neglects friction. In fact, it is only on the Cartesian plan of beginning by isolating different aspects of a problem that a final comprehension of it as a working whole can be attained. The method involved in Newton's second law of motion, of treating co-acting forces as capable of independent consideration, is applicable in a far wider sphere than that of Dynamics. It is in fact almost the only way in which the limited capacity of man can be rendered capable of grappling with complicated problems.

I am indebted to Professor Foxwell for several suggestions, and I am not the first nor shall I be the last writer who has had cause to appreciate the benefit of his magnificent Economic Library, which is to find a home in the new buildings of the University of London.

H. CUNYNGHAME.

134, Cromwell Road, S.W.,
June 14, 1904.

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ERRATA

P. 51 l. 14 *for* of *read* is

P. 65 l. 18 *for* is *read* was

P. 94 l. 2 *for* R′ *read* R

P. 118 l. 24 *for* the laws hardly obey *read* the curves hardly obey

CHAPTER I

INTRODUCTION

In this book I propose to treat of the application of geometrical methods to the study of the theoretical laws of demand, supply, and exchange of commodities.

It is perhaps needless to reiterate what was first pointed out by Descartes, that sciences involving the consideration of quantities of necessity have a mathematical side. For Mathematics is the science of quantities.

It is, however, not necessary nor desirable to be always dragging in Mathematics where they are not wanted. Cookery is a quantitative science, yet we can get our dinners prepared without mathematical calculations. Our houses are built with no more Mathematics than is involved in the rule of three.

I therefore deprecate the unnecessary use of Geometry, Algebra, or mathematical symbols. It is only when they really aid abstract thought or practical calculation that they are useful.

But there appears to be an undue prejudice against their application in the field of pure economic science, and this prejudice it ought to be the duty of every one who finds geometrical treatment useful to endeavour to remove.

It is often very difficult clearly to indicate the author of a particular theory, and to do so may interrupt the thread of the argument. I do not, therefore, propose to deal with the history of this mode of treating Political Economy. A bibliography of books on mathe-

matical methods is to be found in Jevons' *Theory of Political Economy* (3rd ed. 1879). Some writers, however, deserve special mention. By far the most important of the earlier works is that of Cournot, who in 1838 wrote a treatise called *Recherches sur les principes mathématiques de la théorie des richesses.* In the fourth chapter of this book he clearly expounds the law of diminishing utility, and shows that the price which would be offered for an article depends on the utility of the last increment exchanged as compared with money.

The doctrine of final utility was clearly seen by W. F. Lloyd, *A Lecture on the Notion of Value* (Oxford, 1833, published 1834); and by Dupuit in *De la mesure de l'utilité des travaux publics* in 1844. I do not see that Gossen, in his *Entwickelung der Gesetze des menschlichen Verkehrs,* published in 1854, carried the matter any further, or that Richard Jennings, in the *Natural Elements of Political Economy* (1855), expounded anything that Cournot had not already seen.

In 1860 the doctrine of final utility as governing exchange values was discovered, or rather re-discovered, by Jevons, who read a paper on the subject in 1862 before Section F of the British Association at Cambridge. This paper was published in 1866 in the journals of the Statistical Society (Vol. XXIX, p. 282), and formed the basis of a treatise by him in 1871.

In 1870, however, in Grant's *Recess Studies* a paper was published by Fleeming Jenkin, which set out the demand and supply curves almost in their present form, followed in 1871 by a paper in the *Proceedings of the Royal Society of Edinburgh* on *The Principles which*

Regulate the Incidence of Taxes. In this paper the effect of taxation was shown, and the author set forth and explicitly claimed as his own the use of areas to show what was subsequently termed 'consumer's rent.'

Meantime, however, Professor Marshall had been lecturing, and in 1870 had shown to his students the demand and supply curves in a form very similar to those set out by Fleeming Jenkin. In 1879 he printed privately and circulated a paper on *The Pure Theory of Domestic Values,* which set out the whole method in detail. He did more than this, for in the same year he devised a new form for the curves for international exchanges, which he calls the symmetric form, and which was fully explained in a privately printed paper.

Professor Walras next attacked the problem. His first published work seems to have been in 1874. It is analytical in form, but there are some diagrams, and the author suggests some interesting forms for the demand curves. In 1889 appeared a treatise by Auspitz and Lieben. The foundation of this theory is the form of curve proposed, as I believe, for the first time by Marshall in 1879. But the authors, though they mention the names of Cournot, Dupuit, Gossen, Jevons, Walras, as also those of Menger, Wieser, and Launhardt, make no mention whatever of Marshall.

It is to be presumed that these gentlemen did not know of Marshall's previous work, in spite of the fact that copies of his memoranda had been sent to Austria. Whether, however, they did or did not know of it, we may trust that in any future edition they will give that just recognition to their precursor that has been rendered to him by other writers in England, Italy, and America.

Professor Edgeworth has also contributed a number of articles of a mathematical character. The first of these is *The Rationale of Exchange* (1884). He has followed them up by a series of papers extending to the present year. His *Mathematical Psychics* is a characteristic and interesting work (1881). See also his address to Section F of the British Association *On the Use of Mathematics in Political Economy* (1889), *Journal of the Statistical Society*, 1890. In 1892 the theory of the satisfaction of a number of contemporaneous wants was treated in the most ingenious manner by Dr. Irving Fisher, who proposed a new form for the curves. Mr. Wicksteed has also written an alphabet of Political Economy which is interesting.

My own contributions are two papers. One was printed privately in 1886, and in the *Economic Journal*, 1892. The full effect of the suggestions then made has not I think been appreciated, owing perhaps to my very terse and meagre mode of exposition. Another article on foreign exchanges was contributed by me to the same journal in 1903.

The most complete bibliography of mathematical works on Political Economy is that compiled by Irving Fisher, appended to the excellent translation of Cournot by Bacon (1897).

I hope allowance will be made for the fact that the curves I have given by way of illustration are rough and on a small scale, and only intended to illustrate methods. In order to make them of statistical value, explanations would have been necessary which are foreign to the purpose of the book.

CHAPTER II

THE USE OF GEOMETRICAL DIAGRAMS

It is of great importance to form a clear idea of what Mathematics can do for us as an aid to Political Economy, and what it cannot do.

The writer who has best defined the scope of the application of Mathematics is, I think, Professor Edgeworth. In a paper published in 1890 in the *Journal of the Statistical Society* he deals with the whole subject, and it is a pity that this interesting essay is not more easily available; and in various articles in Palgrave's *Dictionary of Political Economy* further observations on the same subject written by him will be found. See also remarks in Cournot's book, 1838, Chapter IV, and in Professor Marshall's *Economics of Industry.*

Wherever any science involves the comparison of quantities, or the laws which govern the ratios of quantities to one another, then we may be almost sure that Mathematics, which is essentially the science of quantity, can with advantage be introduced. We use Mathematics daily and habitually. Every time, for instance, that we think of the size of an army as compared with the population of the country from which it is drawn, we are performing a mathematical operation. No salesman can put a price on his goods nor calculate discount without arithmetic.

In consequence, those who attempt to dispense with Mathematics are at a great disadvantage.

The picture that John Stuart Mill's chapter on Foreign Exchanges presents, of a man trying to think

out mathematical problems without the use of adequate symbols, ought to be a lesson to those who despise mathematical methods of reasoning. Objections to the mathematical treatment of Economics seem to rest on several grounds. Many apparently dislike it for the same reason that they dislike motor cars, phonographs, typewriters, and other modern inventions. Some, again, seem to be unable to think in general terms, or to put symbols for things.

If the necessity for using rules arises they learn them by rote, or else invent some form of *memoria technica* like the barbarous Latin rhymes that were formerly used to solve logical problems.

Others regard mathematical reasoning as some curious unnatural method of using the intellect.

Professor Cairnes seemed to think he had demolished Jevons' mathematical theories when he challenged him to produce any propositions discovered by the mathematical method which was not discoverable by what he called 'ordinary reasoning.' The very challenge showed that he had a very imperfect idea of Mathematics.

For Mathematics is only 'ordinary reasoning,' assisted by a shorthand mode of expression that enables a proposition to be put in a line and visible in one glance of the eye rather than spread over ten or twelve pages of print; and the methods are so simple and powerful as to be worth an effort to understand, if it were only for the pleasure of widening one's field of knowledge.

It is, however, the language of Mathematics that is so trying to many minds, and those too of a very powerful order.

Of course if mathematical formulae are to be used like magical incantations, and we attempt to grind out results by means of rules like playing upon a pianola without the least idea of music, then Mathematics is certain to become a snare. But if the science is used with thorough understanding, it becomes an engine of great power.

It is true that in many text-books those who profess to teach mathematical proofs have grossly abused the system, and rendered complex by means of symbols that which was simple and clear without them. Even Galileo complained in 1631 that he saw 'young men brought together to study to become philosophers who, though furnished with a decent set of brains, yet, being unable to understand things written in gibberish, assume that in these crabbed folios there must be some grand hocus-pocus of logic and philosophy much too high for them to jump at.' 'I want,' he says, 'such people to know that just as Nature has given eyes to them as well as to philosophers for the purpose of seeing her works, so she has given them brains for examining and understanding them.'—*Dialogues.*

The use of Geometry as an aid to Economics and Sociology is by means of diagrams to sum up and present to the eye a number of scattered facts, which when placed in words and figures are not easy to group in the mind, and to employ these diagrams in the solution of economic problems and the proof of economic theorems.

In the application of Geometry and diagrams to Political Economy all that is done is, to draw pictures visible on paper to represent concepts. When we are merely thinking of a quantity of goods, or a sum of

money, or even the ratio of a sum of money to a quantity of goods, such pictures are not of much use. But when we begin to try and form a concept not merely of a ratio, but of a group of ratios, and compare this with another group of ratios,—or still more, when we try and impart these concepts to others,—then the notation of Mathematics becomes invaluable, as a language wherein to express not merely these relations, but others more complex still.

These curve-diagrams are of two kinds, which must be carefully distinguished one from the other, because their nature and uses are very different. The use of the first kind is to present in a visual form a series of facts. An example of this is a common barometer curve, like those made by recording barometers, in which the height of the mercury is marked on a drum caused to revolve uniformly by clockwork.

This form of diagram is so useful that it has been adopted by all Government departments for setting out national statistics. An example is to be found in the diagrams in Fig. 1.

The method of constructing this figure has been, to raise up vertical lines for each year, of a length corresponding in each case to the number of lunatics in that year. In the adjoining table, Fig. 1, the years are shown along a horizontal line from 1859 to 1903. From a base line A B a line for each five years is drawn, and along it upwards is measured a length corresponding to the number of lunatics in the United Kingdom in that year. Thus, in 1859 there were 38,000 lunatics of all classes; in 1868 there were 52,000 lunatics, as shown by the length of the line C D, which represents 52,000 on the scale, the size of which is taken in some convenient

way to suit the space which the diagram is to occupy. The alarming and steady increase of lunacy is shown by the line, which moves steadily upwards till in 1903 we have 110,000 certified lunatics, as opposed to 38,000 45 years previously. The shape of the line also shows

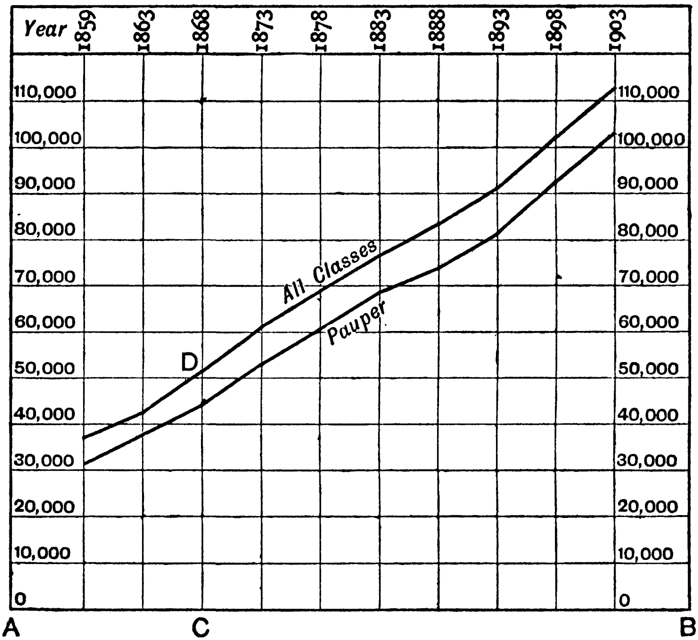


FIG. 1.

the rate of increase, which is actually increasing during the last ten years. Of course the vertical and horizontal scales for years and number of lunatics can be taken in any convenient way. In this scale, horizontally we have about sixteen years to the inch. Vertically an inch represents about 40,000 lunatics. But we might have adopted different scales if it had been more convenient.

It is to be observed, too, that there is no necessary relation between the scales; we might shorten or lengthen either of them independently of the other. The diagram would have a different shape, but it would still show a similar line. Thus, for instance, Figs. 2 and 3 show the same curve as is shown in Fig. 1. In Fig. 2 the vertical lines representing the number of lunatics have been shortened. In Fig. 3 the horizontal line representing the years has been

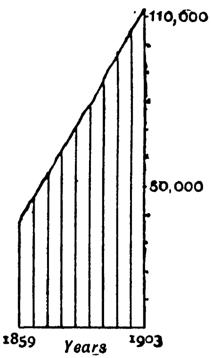


FIG. 2.

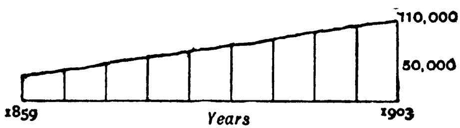


FIG. 3.

shortened. As a result the shape of the curve is not nearly so well shown as when a more convenient scale is adopted.

Where the variations are small as compared with the numbers, it might be well to take the base line so far down that it could not come on the paper. For instance, the number of paupers in the United Kingdom in winter for every 10,000 of the population, and for the years 1888—1903 is given in Fig. 4. It has varied during these years from 282 to 248 (*Ninth Abstract of Labour Statistics* [Cd. 1755], 1903, p. 223). The true

base line from which the vertical lines the tops of which form the curve have been measured, is $4\frac{3}{10}$th inches below the base line A B.

To have put this on the diagram would have been a mere waste of paper, so a start has been made at 230

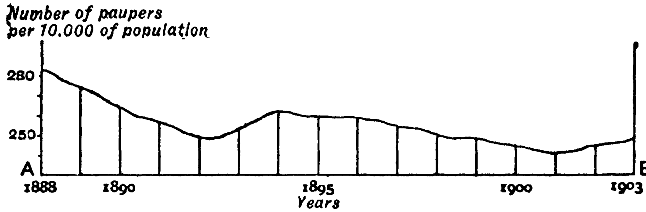


FIG. 4.

paupers per 10,000 of the population, which is lower than the lowest figure ever reached during the period. This figure therefore shows how in drawing diagrams not only the scales but also the base lines may be modified to promote clearness. If the diagram had

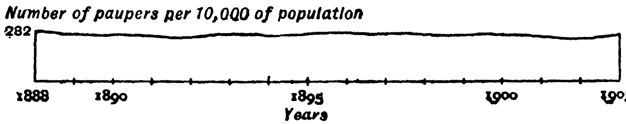


FIG. 5.

been drawn from A B as a real base it would have taken the form shown in Fig. 5, and the curve would have been so nearly flat as to be much less instructive. The sharp rise, for instance, in the year 1892 (a year of falling wages) would have escaped attention.

Thus, therefore, Fig. 4, in which each 100 of paupers per head of population is represented by two inches of vertical distance, produces a far more legible curve

than Fig. 5, in which the vertical scale is twenty times smaller, though in order to secure this advantage it has been necessary to take the base line of Fig. 4 below the base of the diagram.

The next group of curves, Fig. 6, represents a series put together for the purposes of comparison.

Comparative Curves.

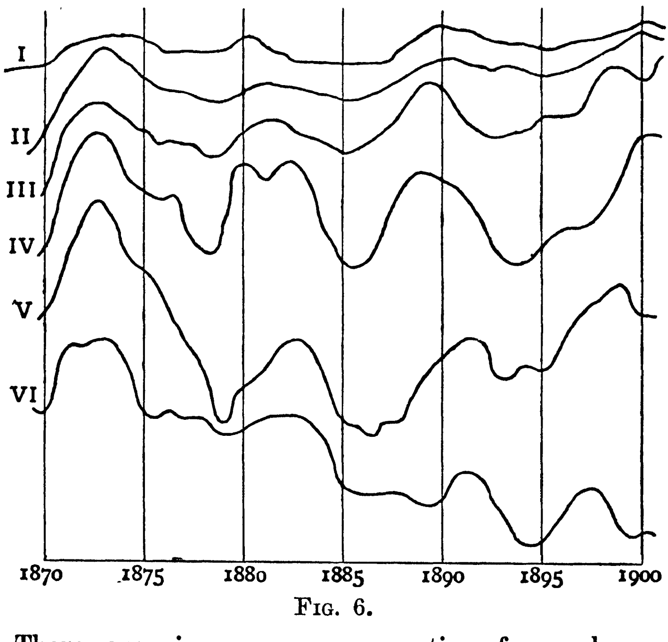


FIG. 6.

There are six curves, representing for each year from 1870—1901:

I The price of coal.

II The average money wages of the working-classes.

III The banking business done per head of population.

IV The total money value of imports + exports per head of the population.

V The number of marriages per head of the population.

VI The price of wheat.

The sources from which these curves are derived are as follow:—The price of coal is furnished by yearly Government returns issued by the Home Department. The rate of money wages of the working-classes is given in Table XIX of the Memoranda issued by the Board of Trade with reference to the question of Fiscal Policy in 1903 (Cd. 1761). The banking business done per head of population, the foreign trade, and the marriages will be found in the annual report of the Registrar-General for 1903 (Cd. 1230.) The price of wheat is also given in the same publication, and in greater detail in the tables of wholesale and retail prices issued by the Board of Trade 1903 (321). The same figures are also given in the 49th Annual Statistical Abstract of the United Kingdom, 1902 (Cd. 1239).

In drawing these curves it has been impossible to adopt the same vertical scale for each of them. The base lines are therefore put at arbitrary distances down. Otherwise the curves could not have been got into so small a diagram. Thus, for instance, the average price of coal in 1870 at the pit's mouth was 5s., in 1890 it was 8s. Hence the base line of the top curve is very near the curve. On the other hand, the base line of the marriage curve No. V, is about 13 inches below the base line in the figure.

But this figure makes it very easy to trace the rise and decline of each series of quantities; and more, it enables us at a glance to see whether these phenomena are connected.

On looking at the curves we shall see that a

depression or elevation on any one is accompanied by a similar depression and elevation on the others. In certain years money wages are good, at the same time the prices of wheat and coal go up, exports and imports increase, banking is brisk, and marriages become more frequent. These are all signs and effects of what we know as periods of good trade. The interdependence of the factors is admirably illustrated by the diagrammatic method.

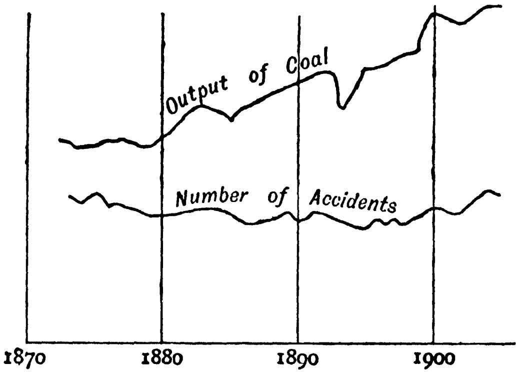


FIG. 7.

Or again, take the curve of mining accidents, Fig. 7. Here we see that though the number is not increasing in proportion to the output, yet the yearly averages follow to a certain extent the rise and fall of output of coal. It was these curves that first drew attention to the fact that when the output of coal rises, for a time less skilled labour is drawn into the trade and accidents result.

It would be easy from the Blue Books to fill pages with these interesting curves, to show the effect of good living and easy times upon the birth-rate of

Great Britain, to trace the fall of pauperism and the rise of lunacy.

Enough has been indicated to show the use of such curves as are employed to group and arrange facts.

The curves we are to deal with in this treatise are, however, of another nature. Their object is not to group and present facts but to represent laws. While these curves which we have been examining are concrete, those which we are about to consider are abstract. And as accuracy of detail is the essence of a fact-curve, so accuracy of conception is of the essence of a law-curve.

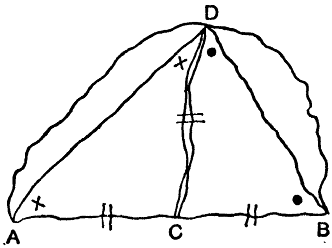


Fig. 8.

If we were calculating by the graphic method the roof-angles and beams of a house, an accurate drawing would be necessary; but a problem of Euclid is seen to be true, even when scrawled in the sand. Its truth does not depend on accuracy of drawing at all. The drawing merely stands for a group of concepts, and holds together not a collection of concrete facts but a group of abstract ideas.

Suppose, for instance, that we had to demonstrate the proposition that the triangle inscribed in a semicircle with the diameter for a base is right-angled. Figure 8, which is all out of drawing, proves it as conclusively as the most accurately drawn one.

The reason is that the proposition is abstract, and so is the demonstration.

And I shall have presently to show that inasmuch as the pure science of Economics is an abstract science, the curves which we shall have to use are of this abstract character.

Before, however, we deal with their shape it will be useful to give a brief explanation of curves in general. A curve has always magnitude and direction; that is to say, it has a certain shape, and a certain size. When

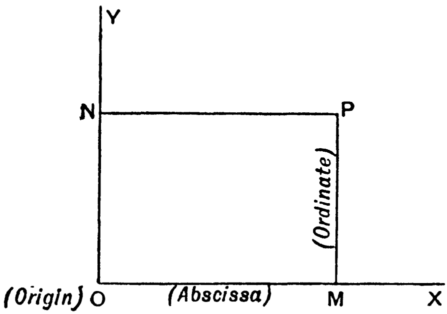


FIG. 9.

these are known, the curve can be drawn. But the shape and size of a curve have to be referred to some fixed bases. The most usual method, and the one which will be adopted here, is that adopted in the diagrams already given; viz. by a reference to rectangular axes of coordinates. By the axes of coordinates are meant the base line O X, and the line of height O Y. The point O is called the 'origin.'

If we know the perpendicular distances, P M and P N, of any point P from these two lines, the point P will be fixed; and if we had a table of the distances of a series of points from the lines O X and O Y, then every one of

these points could be plotted down on paper from the table. O M is usually called the 'abscissa' of P, and M P is called the 'ordinate' of P. In relation to one another they are called co-ordinates; and O X and O Y are called the 'axes of coordinates.' These names will be used in future.

In any table of statistics the figures in one column can be treated as abscissae, and those in the other as

Age.	Expectation of further life.
Years.	Years.
0	40·9
10	47·4
20	39·9
30	33·3
40	26·7
50	20·1
60	13·9
70	8·7
80	5·1
90	2·9
95	2·2

ordinates, and thus a series of points could be put down on paper to represent the table. The points so laid down might perhaps give us a series of points dotted about anyhow, like the stars in a constellation. On the other hand, when we came to plot out the various points we might find that they ranged themselves in regular order. In the table given above the figures in the right-hand column denote the expectation of life of a person of the age shown in the column on the left. Thus, for instance, a person fifty years old, of fair average health,

may expect reasonably to live twenty years more, and so on.

This table may be expressed by drawing two lines, O X and O Y, at right angles to one another, and putting on the paper a series of dots, such that the distance of each from O Y shall represent a given age of a person, and the distance of the same point from O X shall represent the number of further years that he may expect to live. It is not essential that the scales along

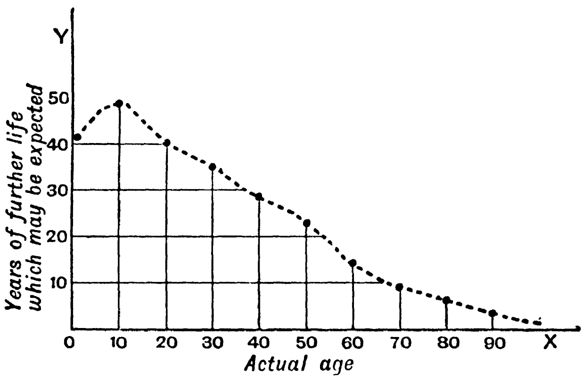


FIG. 10.

O X and O Y shall be the same; we might take a scale of thirty years to the inch along O X, and of forty years to the inch along O Y. But when the curve has been plotted out, the points arrange themselves in orderly fashion, not merely at random.

From this we are able to see at a glance that human life follows on the average some regular law. This law no doubt depends on the mysterious decay of the cells of our bodies, which change and deteriorate with wonderful uniformity. The period of infancy has

dangers. A boy who has lived to be five years old (like a dog that has had distemper) has a better expectation of residual life even than his younger brother. But when he gets to ten years old, then each year his expectation of life declines, until when he is sixty he cannot reasonably expect to live for more than fourteen years longer.

The curve presents this in a simple form. But it shows us more. It shows that the expectation of life is not simply dependent on the age in arithmetical proportion. As the curve goes on it bends a little, being convex to the axis of X. This shows us that as a man's life goes on, his chances of attaining a given period of old age improve. Each rock passed renders a longer journey more probable. When, however, he attains ninety years he may be said to hold life on a very precarious tenure. If life were to be represented by the sand running out of an hour-glass, there would have to be a little stream running into the glass, and adding to the contents of the reservoir, to represent the increased longevity as age advances.

It may be necessary sometimes to modify curves, by increasing or diminishing the ordinates or abscissae by some fixed amount, or to multiply them by some quantity or scale of quantities. Thus, for example, Fig. 11, curve A, shows the total money value of exports from the United Kingdom for the years from 1870 to 1902. If from the total exports we deduct each year the exports of raw material, we obtain a curve B, showing the exports of manufactured materials. This table shows that during the last thirty years the money value of the exports of the United Kingdom has fluctuated, but that the proportion of raw material

exported to manufactured material has remained steadily in the ratio of about 1 to 11.

Thus then we see that curves may be added or subtracted, and the results shown on other curves.

But we may in suitable cases desire to multiply or to divide the ordinates or abscissae of one curve by those of another.

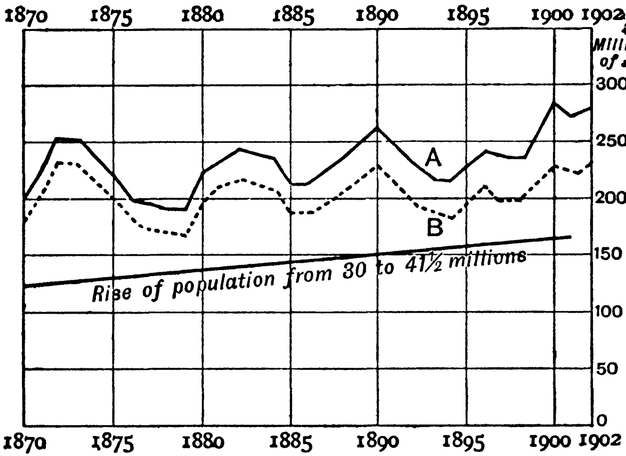


FIG. 11.

The following table, Fig. 12, is a case of division. Curve B gives us the number of men employed under- and over-ground in the coal mines of the country from the year 1885, and curve C gives us the total output of coal for each year. By dividing the ordinates of curve C by those of curve B, we obtain a curve A, showing in each year the output per man. In this case,

the ordinate of A × the ordinate of B = the ordinate of C.

And it is to be noted, that the ordinates here have

been drawn on entirely different scales, as shown by the marginal scale numbers. Note how the sudden drop in output in 1893 has been accompanied by a slight fall in the numbers employed, and a large fall in the output per man. As a matter of interest I have added a curve D, that of the average weekly wages per man. This shows that real wages are now 50 per cent. higher than they were in 1885, while the output per man has decreased by about 10 per cent.

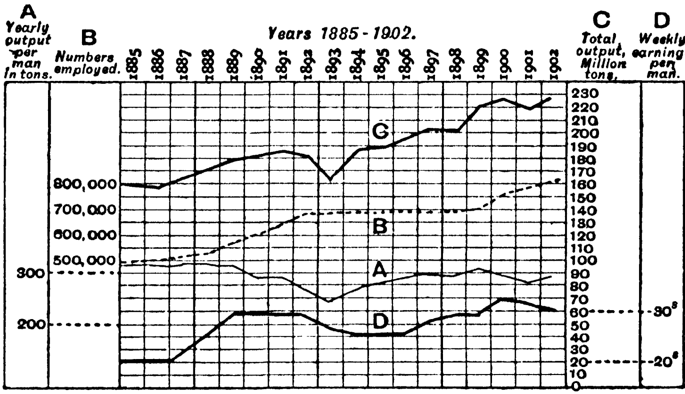


Fig. 12.

This increase of the labour-cost of getting coal has been partly met by increase in efficiency, partly by a rise in price of coal at the pit's mouth from 4*s.* 10*d.* in 1886 to 8*s.* 2*d.* in 1892; that is to say, a rise in average price of 80 per cent. The curves show that in England the present output rate is 282 tons per man per annum.

Another diagram, Fig. 13, illustrative of division, may be given. Curve A represents the general average rate of rise in money wages since 1880. But the mere rise

in money wages is not a sufficient criterion of the real wellbeing of the working-classes. It needs to be corrected by dividing the money wage by the average rise and fall of prices, so as to get at the real purchasing value of the earnings. Curve B is the curve of the rate of general price fluctuations, taken from the Board of Trade yearly tables of prices (Brown

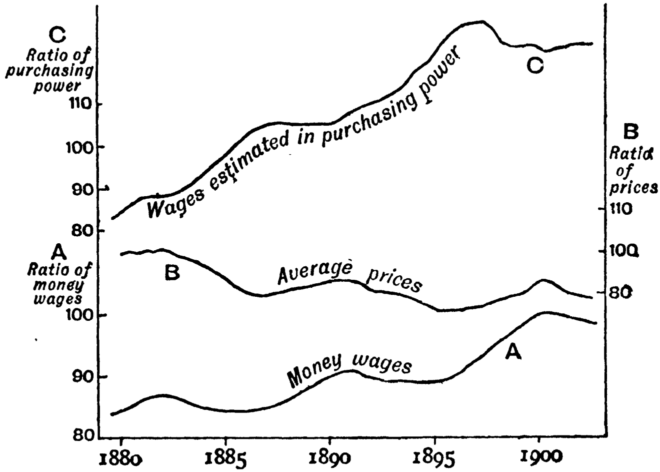


Fig. 13.

Book, 1903, No. 321). If A is divided by B, a curve C is produced, which represents the average rate of wages computed in such goods as are usually purchased by the working-classes, and is thus some measure of their gain in prosperity. It will be seen that in twenty years the average money wage has gone up nearly 20 per cent. The purchasing power of money has in the same time gone up about 25 per cent., resulting therefore in a real increase of the reward of labour to the wage-earning classes of 50 per cent. The

curves A, B, and C show the rates of the various fluctuations; the scales for the ordinates are taken in an arbitrary way, so as to make the curves fit conveniently into the diagram.

Having thus given some examples of the addition, subtraction, and multiplication of ordinates of curves, we may next proceed to illustrate the meaning of the areas marked out by them.

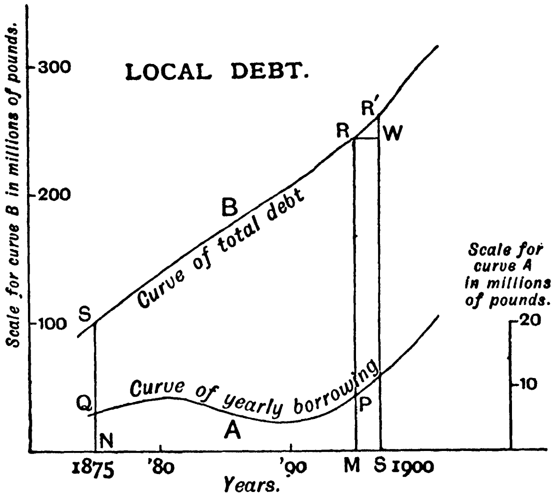


FIG. 14.

Suppose we construct a table marking the years from 1875 onwards along the axis of X, and drawing ordinates upwards such that on a scale each ordinate showed the yearly sum borrowed by the local authorities of England and Wales. This would result in a curve A, Fig. 14, showing that the amount borrowed yearly had varied from about eight to twenty-three millions (exclusive of repayments). Now suppose we know this, but want to know what is the total indebtedness at any time. It is

obvious that the total debt incurred during any period of years is got by adding together the debts contracted during those years. This will be represented on a diagram by the area of the curve. Thus the indebtedness incurred during the years from 1875 to 1895 is represented by the total area P M N Q, that area being made up of all the ordinates from N to M.

In 1875 the local indebtedness was 92·8 millions. In 1895 it was 235·3 millions. Thus, in the twenty years it had increased by 142½ millions, represented by the total area of borrowings = P M N Q. We may represent the total indebtedness at any time by another curve, B. This curve is therefore such that any ordinate M R represents the area M P N Q. Of course we must start from proper points. Thus, if we were to consider the debt as beginning in the year 1875, the point S would be on the base line.

Hence then, whatever form a curve may have, say O T Q in the adjoining Fig. 15, we can always draw an area-curve, or 'integral' curve O P, such that an ordinate P M of that curve shall represent the area O M R T O of the curve originally taken. There are instruments with which this can be done, and which are in considerable use among surveyors.

The use of these integral curves will be seen hereafter, when we come to deal with the total amounts paid for given amounts of commodities sold at various prices.

Returning to Fig. 14, the rate at which the borrowing is going on can also be seen from the curves. For suppose that the debt amounted to R M, or about 240 millions in 1894, and that two years afterwards, in 1896, it amounted to S R′, or about twenty millions more; then that twenty millions of increased debt is repre-

sented by R′W, the amount by which the ordinate has increased. The ratio of this to two years, which period is represented by the line RW, therefore represents the rate or pace at which the debt is increasing. It is a measure of the steepness of the curve of ascent of debt. That ratio in geometrical language is called a tangent, and therefore the tangent to the curve at R is a measure of its steepness, and hence of the rise of the debt. Hence the tangent of the angle of inclination of a curve always measures the rate of increase

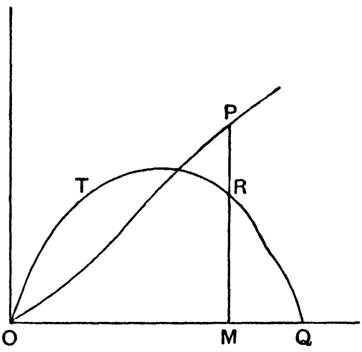


FIG. 15.

of the ordinate as compared with the abscissa. Thus in 1900 the amount borrowed was abnormally large. This is shown by the increased steepness of the curve at the point corresponding to that year.

In this case it measures the rate of borrowing. Looking back at Fig. 12, the curve D shows that although the weekly wages of a miner was not so great in 1887 as in 1900 the rate of increase between the years 1887–1889 was greater than at any subsequent period. The rise in output shown by curve C was greatest in 1898. The tangent of the angle of inclination

of a curve is always therefore an important factor, and can be very clearly shown in a diagram. Thus in

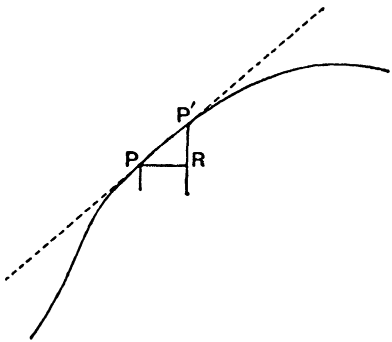


Fig. 16.

Fig. 16, the tangent of the angle P′PR, or in other words, the ratio of P′R, the increase of the ordinate, to PR, the increase of the abscissa, is a measure of the steepness of ascent of the curve at the point P.

Year.	Number of thousands of emigrants.	Totals from commencement of period in thousands.
1893	209	
1894	156	365
1895	185	550
1896	162	712
1897	146	858
1898	141	999
1899	146	1 145
1900	169	1 314
1901	172	1 486
1902	206	1 692

Another illustration of an integral curve is afforded by the table given above of the emigration of British-born

subjects from Great Britain for the last ten years. The totals are added up in the third column, and form the ordinates of the dotted curve of total emigration during the period.

It is noteworthy that the emigration is going on increasing in spite of the rise in money wages and the tremendous fall in birth-rate that is so marked a feature

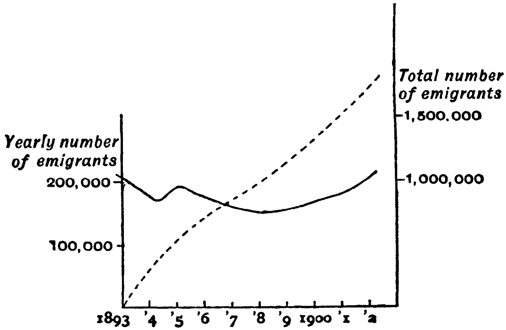


FIG. 17.

of late years in Great Britain. This emigration must be a terrible drain on a country of only forty-four millions of inhabitants. It is to some extent replaced by alien immigrations of Jews, Poles, and others.

The curve formed by the figures in the second column is shown by a continuous line; the curve formed by the figures in the third column is shown by a dotted line, Fig. 17.

CHAPTER III

DEMAND CURVE

ENOUGH has now been said to illustrate the general character of diagrams.

Those which have been given are of a concrete character, but in pure Political Economy the curves employed are usually and principally of an abstract character. The truth of a theory is made to depend on reasoning, rather than on experiment; it deals with proofs rather than with results. Hence the curves will in general be abstract, instead of being concrete like those I have given above.

But, although the curves are abstract, and do not represent concrete curves except in a few instances, it is possible by the geometric method to indicate their general character and tendencies in very useful ways.

I now propose to show how they are applied to the theory of exchange values.

It will of course not be expected that I should set out the whole theory with the minuteness and the qualifications that are to be found in general text-books.

All I shall do is to state the theory sufficiently to show how diagrams may be used to explain and support it.

The problem is, to ascertain the general character of the curves that regulate the rates of exchange of commodities.

In the first place, we must assume that this is done in a market, where transactions become known, and in which those who frequent it are actuated by a desire to get as much as they can and to give as little.

Of course this does not represent real life, any more than an engineer's calculations respecting a timber beam represent the real strength or stresses. But neglecting ignorance and fraud on the one hand, and putting out of consideration custom and benevolence on the other, it is fairly true that in a market trade motives predominate, and that out of the clash of competition market prices are evolved.

Market transactions take place between buyers and sellers. The buyers demand, and the sellers supply. Of course this relationship is reciprocal. A man who demands goods must also supply money. A man who demands boots must supply coin. Thus then, when we speak of demand we do not mean that it is disassociated from supply; rather, we mean that with respect to articles demand and supply are really two aspects of the same question.

Let us then first consider men in their position as buyers or demanders of some given article.

There are many things that every man desires; in fact, every one desires almost everything. But mere desires do not come within the province of practical marketing. To me it is a matter of entire indifference whether pictures by Turner sell for £2,000 or £20,000. I appreciate them. Few things give me any pleasure so intense as a picture like 'Crossing the Brook'; but the desire to have a picture like this always in my room is a mere barren aspiration exercising no influence whatever on the market value of Turners. When, however, our desires become so moderated as to fall within the measure of our means, the whole question is altered, and we then become potential buyers. Makers of articles begin to study us. Articles in trade journals

are written upon us and our wants, and how they may best be gratified. And when we begin to buy in earnest, then we become very interesting. Our influence on the market then begins to be felt.

Speculations as to the pleasure given by wealth in general, seem to me singularly useless. It has been urged by some persons that the pleasure given by any given acquired portion of wealth is measured by its proportion to what you have already. Another theorist suggests that the pleasure given by the possession of wealth varies as the square root of its total amount.

But all these speculations are set at naught by a visit to a working-man's home. We know by experience that the respectable artisan is as happy and in many cases happier than the wealthy classes. Besides, these calculations omit to consider that the acquisition of wealth gives far more pleasure than its possession. The first week's enjoyment of an increased income is greater than the enjoyment of any subsequent week. It may be possible, perhaps, to express human enjoyment in mathematical language; but it would be necessary, I think, to treat it as a function, not of two, but of twenty variables. On the other hand, it is quite legitimate to consider mathematically the ratios of the pleasures given by two definite articles, and one of the most important general laws which govern the operations of a buyer is the law of diminishing utility; that is to say, the law of our nature which so regulates the measure of our enjoyments that each successive acquisition is less valued than the one that has preceded it. It is not necessary to give many illustrations of this. Each article is highly valued if we have only

a little of it; all become less useful, and at last positive hindrances, if we have more than we want of them. At length the market becomes 'overstocked' with the article, and ultimately 'glutted.'

If, however, we are to examine the influence upon a market of these diminishing, because partly satisfied, demands for any article, we must have a standard of comparison. The test is always, What will you give for what you want? It is the ratio of what you will accept to what you will give that we are seeking.

Hence then we must always estimate our demands in some standard. Although wandering tribes begin by simple barter, yet as time goes on exchanges tend to become standardized, and cattle, or cowrie-shells, and ultimately the precious metals are adopted as a universal measure.

It would be out of place here to examine the reasons which have conduced to our employment of gold as a standard, or the properties which render gold suitable for the purpose. Money is the universal standard, and in the next succeeding chapters we shall use it as a measure. In a future chapter, however, we shall take a wider view, and examine how far the theories which deal with exchange values in terms of money need modification.

Let us, then, suppose that our buyer (or buyers) are demanding goods and offering money. We may now measure off quantities of goods along the axis of x, and raise up from each an ordinate representing the price that the buyer would give if the number of things represented by the abscissa changed hands. It is very difficult to determine what we mean by 'changing hands.' Do we mean in a year

or in a day? And again, how are the sales to be made? By successive private purchase, so that those who will give most shall be first served, and those who will give less left to the chance of being able to buy afterwards? Such suppositions are possible, but would render the curves quite complex; so for simplicity we must suppose that by the demand for a quantity at a price is meant a publicly-known demand of a group of buyers for a quantity supplied indifferently to every one, and a public supply or sale at

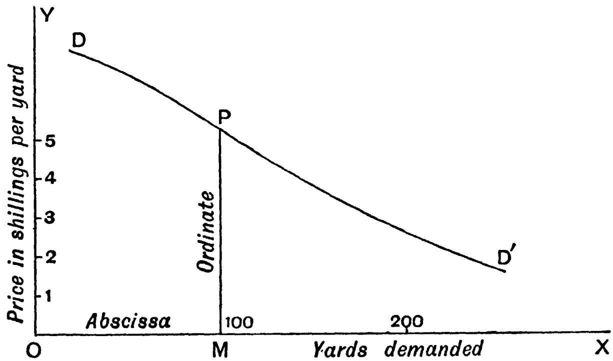


FIG. 18.

a uniform price indifferently to every one by a group of sellers, so that there is no change of price during the transactions.

Thus explained, the demand may be represented by a curve such as that shown in Fig. 18. O X, O Y are the axes, P a movable point on the curve. If P is at any position the abscissa will represent a quantity of (say) yards of cloth, and the ordinate the price per yard that the purchasers would give if they purchased the quantity O M. Thus O M is the quantity which would be bought at a price P M.

It is of the essence of this curve that P M, the price, should constantly diminish, for, as has been said, each successive quantity bought is of less and less value to the buyer, and will fetch a less and less price in the market. It is true that at first the price may not fall much, but if supply only goes on long enough, it must fall at last, and fall very low when the over-supply becomes considerable.

The next point to determine is whether any law governs the rate of descent of the money demand

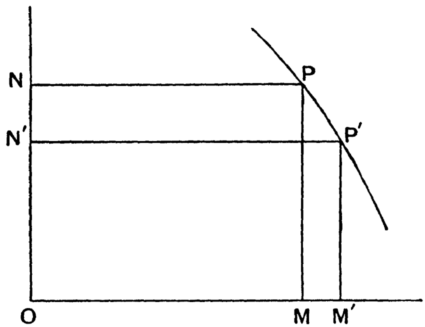


FIG. 19.

curve. An examination of Fig. 19 will show that if at the point P, where a quantity O M of the commodity is demanded, the curve becomes very steep, a point might be reached where N′ P′ M′ O, the total amount of money which the purchaser would give for a quantity O M′, was actually less than N P M O, the total amount which he would give for a smaller quantity, O M.

This situation would not be likely to exist with regard to necessaries of life.

Take, for instance, a case in which demand for a necessary seems of a very rigid character, in the

sense that a rise in price but little affects the quantity which will be purchased. Wheat may be selected. The following table shows the consumption of wheat per head of the population since 1890, and the price of wheat per quarter. The figures are for the total consumption for all purposes, including, for instance, the dressing of calico; but by far the greater portion is eaten as food, and forms nearly one-half of the total nutriment consumed by the people of the nation.

Year.	Yearly consumption of wheat in Great Britain per head of population.	Price in shillings per quarter of 480lb.
	lb.	s. d.
1890	344	35 5
1891	357	33 4
1892	348	26 8
1893	345	25 5
1894	357	21 5
1895	333	24 10
1896	335	28 8
1897	324	36 2
1898	345	26 0
1899	341	26 4
1900	345	27 1
1901	342	28 1

This table shows how little difference in consumption is made by a considerable fall in price. Even if we went back to the days when wheat was 60*s.* a quarter, as in 1877, we should still find that the amount consumed per head but little diminished.

If we assume that the general conditions of demand have not much altered, say in the last few years, then we might venture to treat the figures for those years

as typical, and indicative of the changes which a change in price would make in the national consumption at any given epoch.

Now, looking through this table, we find that a rise of a shilling or so usually produces a shrinkage of annual consumption per head of some seven or eight pounds, but as a rule we do not find that a larger aggregate sum per head is paid for a smaller quantity, except in years where some other cause such as wages come in. The figures are very rough, but it looks as though the total aggregate expenditure on wheat were nearly proportional to the population. This, if true, would mean that the demand curve, though it descended with a steep descent, never became quite vertical.

In some cases, of things the use of which is limited by their very nature, it is possible that the curve might become very steep. One instance is that of an etching, when the artist undertakes to destroy the plate after he has sold a certain very limited number of impressions. Here the value of the print in the eyes of those who buy it depends to a large extent upon its rarity, and the fact that others do not possess it. This fancy for early impressions dates from the time when the art of coating copper plates with steel was unknown, and hence when about 200 plates were all that could be got in a really perfect condition. But though the reason has ceased, the fancy still subsists, and this brings about the fact that purchasers will give a bigger total sum for 300 impressions than they would for 1,000.

Or again, suppose that an Arabic English Dictionary is published, or such a work as the present. It is certain that after a given number of copies have been sold the wants of all those who care for the subject

will have been exhausted, and the demand curve will drop to a vertical line.

With regard, however, to most of the usual commodities of life, such as food, clothing, or furniture, there is always a large unsatisfied demand, which would come into play if the price were only low enough; and in those cases the demand curve would only gently descend towards the axis of X, and rarely become very steep within the range of practical conditions.

Where it is the case that people would not give a less total sum for a larger quantity of an article than for a smaller, this would be expressed geometrically by saying that the demand curve would cut negatively a rectangular hyperbola.

For a rectangular hyperbola is a curve such that if x and y be its ordinates $x y$ is always equal to a constant quantity, say C. By giving C different values, of course a whole group or nest of hyperbolas may be drawn with respect to any axes. And what is meant by saying that the demand curve was not so steep that the total amount paid for a given quantity of goods should be greater than the total amount given for a less quantity of goods is, that the area $N'P'OM'$ must always be greater than NPMO. Now since, by the property of the rectangular hyperbola NPMO is always equal to $N'ORQ$, it is obvious that the line DPD' can never at any point cut a rectangular hyperbola negatively, so as to leave the hyperbola to the left of it as the abscissa increases. In this case DD', as drawn in the diagram, is impossible. For a rectangular hyperbola is such that at any point $PM \times OM$, or $x \times y =$ constant. Another property is that if ST is a tangent at R, SR=RT.

The next general characteristic of a demand curve is, that it shall not cut a vertical line more than once. In other words, that for the same demanded total

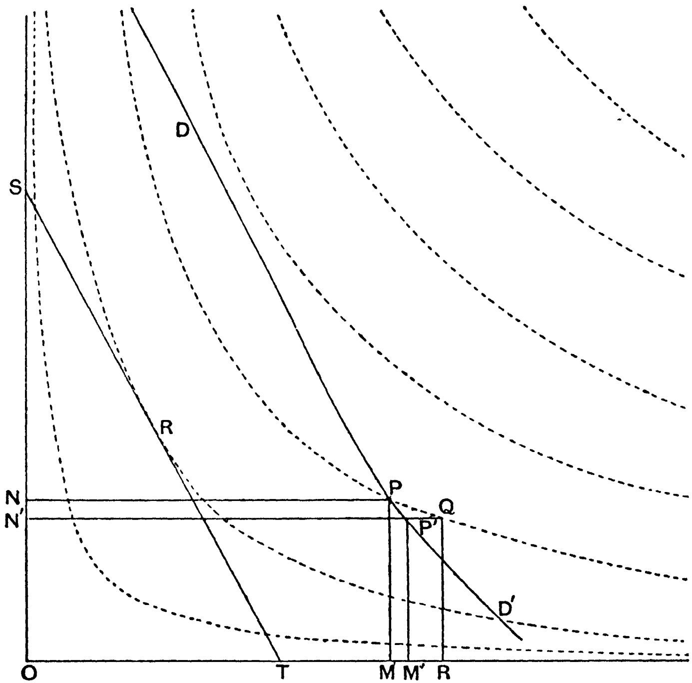


FIG. 20.

amount of an article there shall not be two prices for the last increment demanded.

This assumption is of the very essence of the nature of a market, where in any given state of the demand there will be one price only.

CHAPTER IV

THEORY OF FINAL INCREMENTS

ASSUMING these characteristics for a price-demand curve, we have next to determine what part of that curve will be effective in regulating prices.

The answer to this question involves an inquiry into the general character of the causes which induce a purchaser to part with his money in order to procure an article.

This is usually expressed in a law which is called the law of the relative utility of final increments, and is this, that the determining element as to the last quantity of a thing which is purchased is always the money value to him, not of all or any part of what he has already, but of that last quantity only.

A man who is buying books balances the last new book against the price of it. True, his desire for the book and its money value to him are influenced by the number he has already; but the actual price he will give is measured, not by the value to him of those that he has got, but by the value of that which he wishes to purchase as compared with the money he has to spend. In fact, it is only the parts of the curve about P that influence the amount PM that is demanded. Of course, the books that he already has assist in fixing the point P; and the point P itself is determined, or rather dependent, not only on the books he already has but on the whole conditions affecting his power of using them.

Primarily, however, it is the point P that is important in the demand curve. And the curve itself is simply and solely a curve of final-increment prices. It tells you nothing for certain about the enjoyment he has in what he has got already. It simply tells you that when the price is MP, then a quantity OM will be demanded, and that the ordinates along the whole curve are simply and only the prices of the last portions demanded, when the quantities demanded are those shown by the abscissae, Fig. 21.

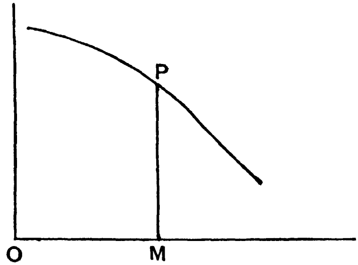


FIG. 21.

From these considerations the general law of economy of expenditure follows, namely, that when a man has a certain sum of money to spend, he will distribute it in the purchase of commodities in such a manner that the utility measured in money of the last increment of each article that he buys is the same.

Thus, suppose that a workman's wife on a Saturday night takes her purse to the market to buy various things she wants. She reflects that she must have at all events bread ; then perhaps she will want some meat, butter, tea, sugar, and cheese ; and then, if her funds allow, she may buy a piece of calico, or perhaps a toy. How will she divide her money? No doubt first she will

buy some bread, meat, butter, and tea and sugar. So far but little perplexity of mind will have arisen. But as the first most pressing wants are satisfied, the difficulty of apportioning her purchases increases. As soon as she has resolved upon the amount of bread, meat, butter, tea and sugar, the question will arise upon what is the next money to be spent? Shall it be more meat, more tea, or shall she see if she may venture on a little calico? That will simply depend on whether any fixed sum such as a penny will be more useful when spent on calico than in any other way. She reflects; she balances a yard of calico at 4½*d.* against another ¼lb. of butter, or perhaps against ½lb. of cheese, and she decides the matter according to the utility of the last increment of each article which she is proposing to buy. Hence, if on a review of her purchases the increment of final utility (measured in money) on any one article appears larger than the increment of the others, she makes it up by buying more of that article, until she gets so much of it that its increment of utility sinks to the general level, and to do this perhaps she sacrifices her desire to buy some other article.

This process may be compared to a sort of weighing in which the turn of the balance does not depend on how much is already in the respective scales, but on the utility-producing value of the last small quantities added on either side.

Another illustration may assist in making the principle clear.

If a crammer were preparing a student to pass an army examination at which, in general, each successive hour's study applied to any given subject produces a constantly diminishing increment of marks, his con-

stant endeavour would be so to distribute the pupil's time that the mark-earning effect of such hour's work should be as great as possible. To this end he would make him spend his time exactly on those subjects that paid best. He would not care whether the pupil was well educated, but would simply distribute his time so as to make the marks gained a maximum; and it is evident that this will be done by putting each increment of study on to the subject, not which he knew best, but on to the subject in which such increment of study would earn the greatest increment of marks.

When this process had been carried on so that each subject had been worked up to its exact level, the time would be scattered among them all, something as a swallow feeds her young. The University plan of making each hour's study of a subject tell not inversely, but directly in proportion to the student's previous knowledge of it, of course upsets this plan entirely by introducing a law not of diminishing but increasing returns.

An elaborate mechanical device to illustrate this law has been contrived by Dr. Irving Fisher, in his thesis for a degree at Yale University in 1891.[1]

I do not propose to give his whole theory, as it is rather complicated, but I can I think give an idea of it by the following illustration. Suppose that the total purchasing power of a man or a community be represented by a certain fixed quantity of water in a jug. The quantity of the water therefore represents so much money. Let him now have before him a number of glasses, each representing a commodity upon which he

[1] *Transactions of the Connecticut Academy*, vol. ix, July, 1892.

is going to spend his money, and let the quantity of water poured into any glass represent the total amount of money expended by him on the commodity which that glass represents.

In order to solve the problem the glasses must be shaped and arranged in a particular way. They must be so arranged that when they stand on a table their brims are all at the same level, and their shape must be so contrived that if a certain measure, say a teaspoonful of water, is poured into any one of them the amount of

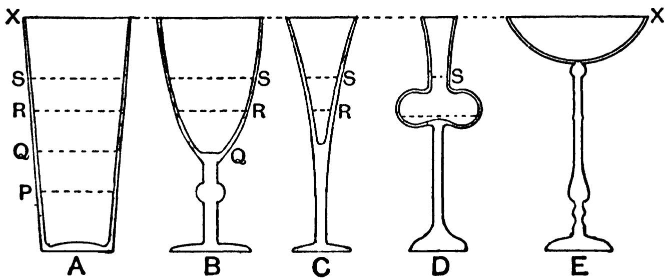


Fig. 22.

pleasure it will give (measured in, or as compared with water, that is with money) will always be proportional to the distance below the brim of the level of the water in the goblet. The shape in each case will therefore depend on the utilities of various quantities of the material represented by the goblet, as compared with money.

Let A B C, Fig. 22, be a number of goblets. Now let the purchaser take his jug of water and with a teaspoon begin ladling it into A. The pleasure each teaspoonful will give is measured by X P where P is the already existing level of the water. At first he will fill

A only. A teaspoonful poured into B would only give X Q of pleasure. When, however, he has got up to the level Q in A, it would be foolish for him to go on pouring more. He can get greater pleasure by paying attention to B and C. And it is obvious that his aim should always be to fill up the deficiency of any glass in which the liquor is more below the level of the brim than the others, because a teaspoonful of water used in this way gives a greater increment of pleasure, or utility, than when used in any other. Suppose, then, that he begins to spend his money on beer in glass A; he adds stout in glass B; then perhaps whiskey in C. As soon as the level R is attained, he gets to port, but after a certain point is reached (represented by S, where the glass contracts to a mere tube) he will not spend much more upon it. Lastly, if his money still holds out, he may spend a little on glass E, representing champagne. Here perhaps his jug of water fails, and he stops. If he gets a legacy, and someone refills his jug, then, if he is a fool, he fills up his champagne glass till it reaches the brim; everything he adds after that gives no pleasure (for X S then has no value). If he still goes on, the goblet flows over and he feels the consequence in pleasure of a negative order, followed by the introduction of another goblet representing physic. But if he is wise, he looks round for other vessels into which he can pour his newly acquired wealth, and perhaps finds that some of them are deeper and will hold more than wineglasses.

Throughout the whole operation, however, the problem always is to put your last teaspoonful where it will tell most; and this is the problem confronting all who have to arrange a budget.

Although the marginal movements of pleasure are in all cases the same, of course the total pleasures derived from the different articles are different. For the *total* pleasure of any one article is represented by the volume of water in the glass. Thus the glass A filled up to R represents a very different total of pleasure from that afforded by the volume of water in glass B filled up to R. But the marginal utilities are the same, provided always that the curves of the glasses have been so made that the distance of R below the level of the brims truly represents in each case their marginal utilities.

Dr. Fisher goes further, and suggests the ingenious plan of connecting the bottoms of all the glasses by tubes to a common reservoir. Then the water need only be poured into the reservoir, and the levels in the glasses will adjust themselves.

Analogies to this are no doubt to be found, when a jug and glasses are to be taken for a whole community. But in practice, in a community, there is not a mere scientific ladling out of a jug into glasses; but a pouring of millions of jugs into millions of glasses, from jug to jug and glass to glass, with a good deal of slopping about of liquid in the process, accompanied by strikes, locks-out, rings, trusts, bankruptcies, frauds, and smashing and overturning of jugs and glasses, whereby the levels of the liquids in the various vessels by no means always present a spectacle of uniformity.

To know how to fill up glasses properly is the art of life, and of good government. It is really the unconscious aim of well-regulated communities. The theory above explained is the real essence of the law of demand.

Of course the demand in any market is subject to daily and weekly variations. It is desirable, however, to ask what we mean by a change in demand, because much confusion may result unless a clear distinction is made between two different meanings that may be given to that expression.

Suppose a person has been paying 2*s.* 6*d.* a pound for tea, and has bought weekly 2lb. If the price went down to 2*s.* for the same tea, he might perhaps increase his weekly purchases and buy 2½lb. each week. The amount demanded would have altered from 2lb. a week

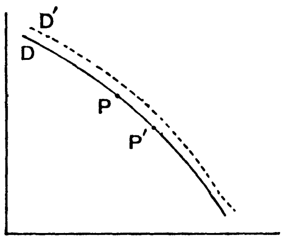


FIG. 23.

to 2½lb. a week. But would his 'demand' have al tered? Not at all. His 'demand' in its true meaning of the general group of amounts he was prepared t give for tea, each amount being dependent on gettir it at a certain price, has not changed. If tea goes bac to 2*s.* 6*d.* a pound, his weekly demanded amount w return to 2lb. It is not the demand that has chang but the supply.

Hence, then, in speaking of changes of demand, must carefully determine whether what we have in r minds is some change of relative utility of tea d money to the purchaser, or a mere change in the amount

purchased without any corresponding change in the value of tea to him. It is only the first of these that is a true change in 'demand.' It would be indicated by a change of the demand curve into some new form or position. The other would merely be a shifting of the point P along an unchanged curve (see Fig. 23). In actual life, demand curves are always shifting with every change of human need or caprice. They gently oscillate like the long ribbons of sea-weed in the ocean. Any picture that can be drawn of a demand curve is only like an instantaneous photograph representing what it was at any given instant. And yet, since these changes of form are very often slow, we may picture them by a line, blurred indeed like the photograph of a tree that has been swayed by the wind, but yet constant enough to be treated for our purposes as representing the demand curve for the time being; a demand curve, composed of a group of an infinite number of demanded quantities, each with its correlative price represented by the ordinate corresponding to the abscissa which represents the amount demanded.

CHAPTER V

SUPPLY CURVES AND FIXATION OF PRICE

WE now have to deal with supply. In this case we do not meet with the phenomenon of gradual satiation and decrease of marginal utility, because the thing demanded by and supplied to the vendor is money, which being the measure of all values is not of a character to produce satiation. We may assume, and in practice it is not far from the truth, that in the ordinary market no group of vendors find that they have too much money. There may be a glut of gold, but not of wealth in general. From this it follows that if we draw a supply curve, such that the amounts that are offered for sale are represented by abscissae along the axes of x, and the prices at which they are offered are ordinates, we are not able to say *a priori* whether the curve will be an ascending or a descending one. It is usual to divide production into three groups, according as the expenses of production (which where there is freedom of manufacture ultimately regulates the prices at which articles are offered for sale) increase, are uniform, or decrease with the amount produced.

The first of these conditions usually obtains in agriculture, the second in hand-industry, and the third in manufacture (a term which originally meant hand-made articles, but which by a curious transformation of meaning now means articles made by means of power-machinery and not by hand).

In agriculture the cost of production usually increases

in correspondence with the amount produced. For increased output involves resort to land that is less and less suitable for crops, and on which consequently more labour and capital must be applied to grow a given quantity.

The agricultural supply price curve will therefore generally be of the form represented in Fig. 24.

It generally rises, but it need not always rise. Economy of production due to larger output might

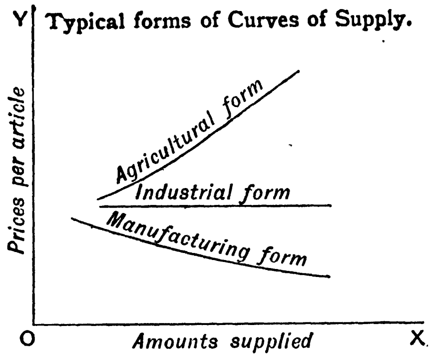


FIG. 24.

make it fall for a time; but inasmuch as increased output calls into cultivation land successively worse and worse fitted for culture, so in general will the curve rise.

For industrial production the supply curve will in general be pretty nearly horizontal. Each article takes as much and no more labour to make than its predecessor. Hand-made boots and clothes to measure are perhaps examples of such a supply.

The most remarkable form of supply is, however, hat afforded by manufactures where each successive crease of production cheapens the cost of producing

the article. We may take the production of a medal as a type of such a supply.

For we may suppose that it costs a manufacturer of medals £20 to produce a steel die, and after that has been made it costs him 5*s.* for the metal and stamping for each medal. If then he produced only one medal the cost would be £20 5*s.* If he produced two medals, £10 5*s.* each. If he produced fifty medals, the cost would be 13*s.* each; and so on.

TABLE OF COST OF PRODUCTION OF A CROWN 8VO BOOK OF 320 PAGES, INCLUDING PRINTING AND BINDING.

Number of Copies Issued.	Price per Copy in Pence.
1 000	18·62
2 000	14·06
3 000	12·56
4 000	11·78
5 000	11·74 [1]
10 000	10·29
20 000	9·71
30 000	9.56 [2]
40 000	9·47
1 000 000	9·31

This example of a medal illustrates the case of an article the supply of which becomes less costly in proportion to the amount produced. The publication of a book is another case.

In order to make the case correspond as nearly as possible to actual facts I have procured from a leading publisher the table given above, showing the cost of production of a book.

[1] Here stereotype plates are introduced.

[2] Here duplicate blocks are introduced.

The results of the table are shown in a curve in Fig. 25.

After the 5,000th copy the cost of plates suddenly increases, and after the 30,000th copy duplicate blocks come in. But these extra expenses are overborne by the steadily decreasing cost of other items. By the time 20,000 copies have been reached, the supply curve is almost a straight line.

As a rule a supply curve, even of the descending

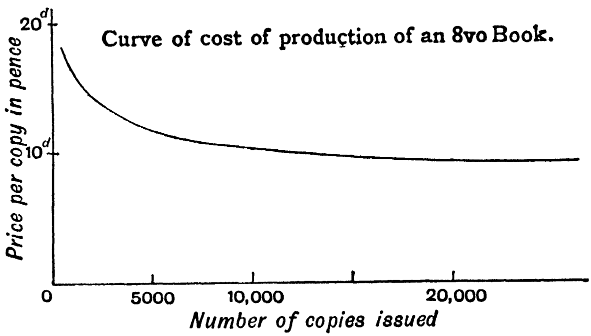


FIG. 25.

order, will not cut a rectangular hyperbola negatively. That is to say, it is not conceivable that in any given condition of manufacture it would actually cost less to produce 1,000 articles than 999. For you could stop when you had made the 999th article; if you did not, then to keep the price up you would have to destroy it.

The case, Fig. 26, in which a descending supply curve most nearly approaches the form of a rectangular hyperbola is in such a concern as a concert. If one expended £200 in hire of rooms, lights, attendance,

and musicians, then if 100 tickets were sold their cost would be £2 each, 200 tickets would cost £1 each, and in fact the cost price of a ticket would always be £200 divided by the number of tickets; that is to say, number of tickets × cost of each is equal to $x \times y$, which is constantly equal to £200. In this case the supply curve would be a rectangular hyperbola of such a character that the abscissa, that is the number of

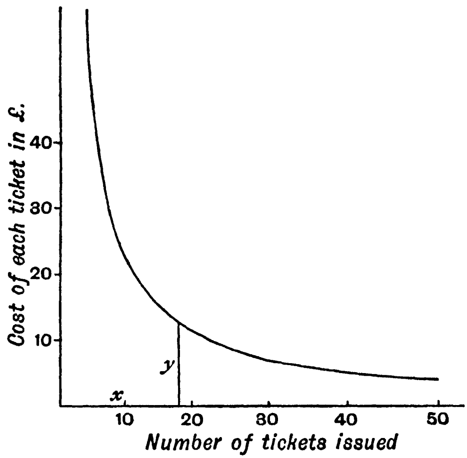


Fig. 26.

tickets issued, multiplied by the cost of production of each, that is the ordinate, would always be equal to £200. An examination of Fig. 30 will show that the curve there has the required character. In every part of it $x \times y = 200$.

But this case will not be a very common one, and a case of actual increase of cost as the amount made was lessened will be almost impossible unless new machinery is employed.

In practice a contractor might well be expected, in the case of an article that could be stamped out or made with increasing cheapness in great quantities, greatly to decrease his prices in proportion as a larger quantity is taken; but I have rarely heard of a contract that provided that less should be paid for M + 1 articles than was contracted to be paid for M articles. And if by the peculiar form of the contract such steps down in price appeared, they would only be small as compared with the totals—the general trend of the curve would be at its greatest falling rate, a hyperbola.

There are, however, circumstances under which the supply curve may descend very rapidly or with a sudden jerk, as when at a certain point of production it becomes profitable to use new kinds of machinery. In this case for a short distance the curve might become a vertical line. But it would soon begin to slope away again, tending towards the horizontal.

We saw that a demand curve cannot be conceived as cutting a vertical line more than once, so as to render possible two different demand prices for the same total quantity of an article. A similar law regulates the supply curve. For otherwise we should have to conclude that when the same amount was produced there would be two different costs of production of the final increment on the part of the same producers, which would be contradictory. There might conceivably be a possibility, by the use of different modes of production, of two costs of production for the same output; but of course the producers would ultimately choose the cheapest, so that in practice not more than one would be found.

Having got our demand and supply curves, no

difficulty now arises in showing that the quantity of an article produced and sold will be fixed by their point of intersection. For if the price which the purchaser will pay exceeds the cost of production, more will be produced; if the quantity made is such that the cost of production exceeds the price that can be got, production goes back to the point at which it is remunerative. And this is what is meant by saying that price depends upon demand and supply. Hence,

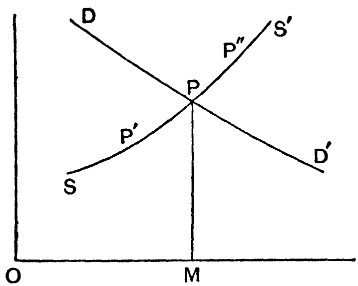


Fig. 27.

in a condition of equilibrium neither more nor less than O M of the article will be made, and the price will be M P (Fig. 27).

1. The money price demand curve of an article has in general the qualifications mentioned above, that it descends.

2. The supply price curve may ascend or descend, but in general, except by jerks, cannot cut a rectangular hyperbola. In other words, no seller will, in normal conditions, deliberately ask more for a smaller quantity of articles than for a larger quantity. He would sooner raise his price and destroy the balance, or else 'dump' it on a foreign nation.

3. Neither curve will cut a vertical line more than once.

The supply curve may cut the demand curve more than once, as in the figure. This would mean that there would be three points, P, Q, R, any one of which would equate the supply with the demand. Of these, P and R would be in stable equilibrium, because any slight over-production which carried P along the curve

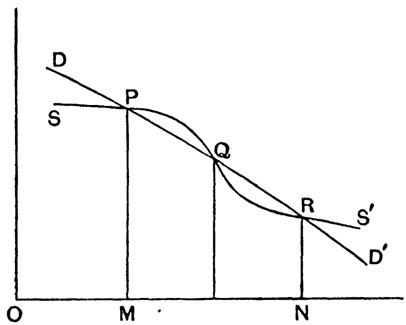


Fig. 28.

to the right would be followed by a loss to the producer, which would speedily tend to bring him back again to the former point of production. But a movement forwards from Q would be attended with profit to the producer, and he would therefore not remain at Q, but go on to R, where again the fixation of prices would be in stable equilibrium.

It follows from what has been said that when once a stable condition of supply and demand has been reached, no change can take place, either in the amount produced, or in the price, except by reason of a change in the demand curve or in the supply curve. By a change in the curves is meant, as has been explained

above, true alteration in the demand, in the sense that at the same price a different quantity is demanded to that which was demanded previously, or that the same amount can be produced at a greater or less cost. In this sense of course supply and demand are constantly changing. For instance, the price of coal has risen in England from an average of 4*s.* 9*d.* per ton at the pit's mouth in 1887, up to 10*s.* 9*d.* in 1890. Had the demand curve remained steady during the interval, of course this would have resulted in a diminution of the amount purchased; but instead of that being the case, the quantity bought when the price was 4*s.* 9*d.* in 1887 was about 150 millions of tons, whereas in 1900 about 220 millions of tons were bought at 10*s.* 9*d.* In the interval, owing to the increased use of machinery, the demand had enormously increased. The curve of demand had moved upwards, parallel to itself, no doubt also changing its shape in the process.

CHAPTER VI

SURPLUS VALUE

THE demand curve being thus considered as a curve of final-utility prices, in which each point marks a price of an increment last obtained, it remains to inquire, when an amount is purchased at a price, what is the utility of any portion to a purchaser as compared with the price he has actually paid for it?

It by no means follows that a man who values a mutton chop at 2*s.* 6*d.* is necessarily obliged to pay that price for it. He may get it for 1*s.* In this case he profits to the extent of 1*s.* 6*d.*

It is usual to treat the demand curve as a measure of these utilities. For it is argued, if a man would have been willing to give a price, P M, for an article, and nevertheless gets it for a price P′ R = Q M, his gain is measured by P Q. There is here, however, an assumption that is not warrantable. The curve means that P M would be the price he would pay if O M only were supplied; it does not mean that P M is the price at which he values the M$^{\text{th}}$ article when O R is supplied. P M is a final-utility price, not an intermediate-utility price.

It is quite true that P M may, as well as being the final-utility price when O M is bought, be also the intermediate-utility price of the M$^{\text{th}}$ article when O R is bought, but it is not a logical necessity. Suppose the article were orchids. Then if only O M orchids were

obtainable at a certain epoch, in a certain market no doubt they might fetch a price P M. A certain limited number of rich purchasers would value them at P M, or higher. But if a successful orchid year occurred and a number of orchids were grown, so that O R orchids were on the market, then the fact that orchids were common might quite possibly react upon the utility of the orchids to the richer purchasers, and cause their utility to be less than P M. Thus, the yearly utility value of the orchids to the richest purchasers

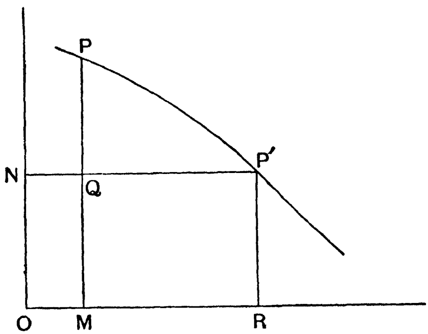


FIG. 29.

might vary from year to year, and yet their demand for and use for orchids be quite unvarying when the prices were the same. The utility of the final increments might be invariable, but the utility of the intermediate increments might vary, and this might occur without any change in demand of the character indicated at the close of the last chapter. Given the price, the amount demanded might always be the same year after year and at all times. The conditions of demand might be quite unaltered, and yet when differences of amounts purchased took place there

might be differences, not only in the marginal prices, but also all along the line in the whole of the articles demanded.

In fact, a change of price might not only affect the amount demanded, but it might also affect the value to the consumer of every article purchased. Inasmuch, however, as the demand curve is only a curve of final utilities, and the changes we are considering affect not final utilities but utilities other than final, there seems

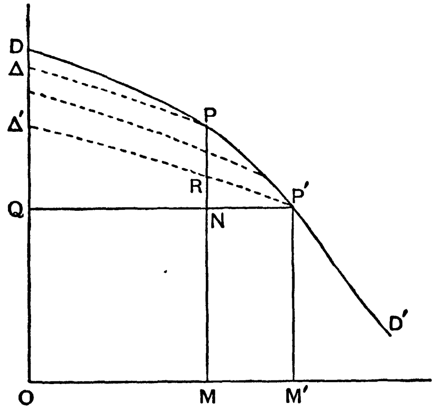


Fig. 30.

to be no mode of expressing this fact except to suppose that corresponding to each point P of final utility on the demand curve there is a whole curve, or group, of intermediate utilities, as shown in Fig. 30.

These curves may or may not coalesce with the demand curve. Often they will, sometimes they will not. Nor are these curves short-period demand curves. They need not move, they may remain fixed as the demand for lotus flowers among the ancient Egyptians; but if they exist, though quite invariable, they will

show that the utility of any given article may vary while the true demand curve of final utility remains absolutely unmoved.

In fact, corresponding to each amount purchased there is not only a final-utility price, but there is a whole group of money values of every single article taken from the first to the last. And as the amount purchased varies, not only may the price vary, but every single value may vary all along the line, as shown in the last figure. And yet all these changes may take place without any true change in the demand curve or its subsidiary curves abovementioned.

In a 'market' the price is always uniform at a given time. Hence, though the utility values of the articles may be unequal, all the articles sold will change hands at the same price. Therefore with a descending demand curve the purchaser has a gain on each article bought except the last. This gain is called by Professor Marshall the 'consumer's rent.' I have never considered this term a very happy one. The word rent is a legal term consecrated by long usage to mean a price paid for the hire of real property, and the amount of which does not always depend on surplus value.

The gain is much more akin to the notion of an 'unearned increment.' The word 'increment,' however, is already devoted to mean small increases of quantity or price. On the whole, I think the words 'consumer's surplus value' best represent what is meant, and as the term is already in use by Professor Marshall and others, I propose to adopt it. The consumer's surplus value, therefore, on the M^{th} article will be R N, Fig. 30, and when a quantity O M′ is bought his total surplus value will be represented by the area P′ Δ′ Q, which

may often for practical purposes be considered as coincident with P′Q D, but not necessarily so.

As in the case of the demand curve we found that each point representing a final-utility value had corresponding to it a special curve of intermediate-utility value, so in the case of the curve of cost of supply of final increments we have also curves of intermediate costs of production. For it by no means follows that because P M is the cost of producing the last increment P when O M are produced, that therefore when O N is

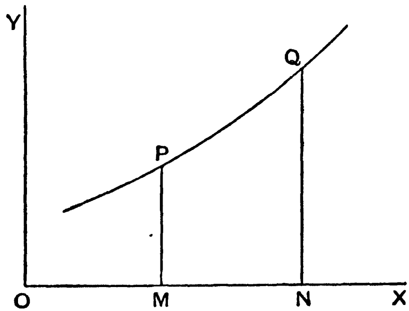


FIG. 31.

produced P M is the cost of producing the M^{th} article. P M by definition depends essentially for its value on the condition that the total production is O M.

It is quite true that where we are dealing with land, the intermediate costs of production when O M are produced will very likely be the ordinates of the curve of cost of final increments. For in agriculture successive lands will be simply brought under cultivation one after another.

But what meaning are we to give to such a statement when we apply it to a supply curve of the descending

order, as in Fig. 32? Can it be alleged that when ON is produced the cost of the M^{th} article is PM? Certainly not; to say that would be to ignore the character of the supply curve altogether, and to assert that the M^{th} article had been made and sold at a loss. In the case of books, for example, it is certain that when the size of an edition has been resolved on, and the cost of production is known, that this cost is the

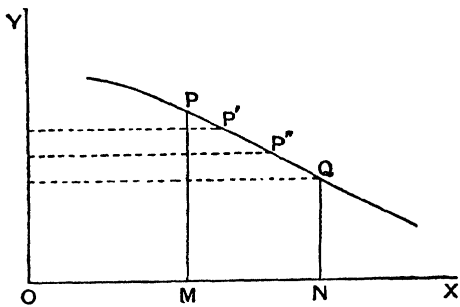


FIG. 32.

same for the first copy as for the last; for in the case of books no one book of an edition can be said to be produced in more favourable conditions than another.

Hence for the supply curve of books we shall require a group of curves, one corresponding to each point in the curve of cost of final production[1].

For the very nature of the supply curve is based on the fact that the curve of cost of final increments does

[1] This was pointed out by the author in a privately printed paper in 1889, and again in an article in the *Economic Journal* in 1892.

not necessarily show the cost of any increments but the final ones. For, as we shall presently see, if it were treated as a curve of intermediate costs of production, we should be confronted with the phenomenon of negative rent in all cases of manufactured articles, and the total cost of production would have exceeded the total price for which the goods were sold—a result which is obviously erroneous.

Nor again, can a line drawn horizontally through P be considered in the case of an article such as books as a 'short-period' or temporary supply curve. It is the only supply curve that ever exists in the case of books that do not go to a second edition, and a very long-period supply curve many authors find it!

This appears to be the only way of explaining the fact that with a rising supply curve you *might* conceivably still have little or no 'rents,' and that the phenomenon of 'rent,' or, as I should prefer to call it, surplus values, may be found in falling supply curves or in horizontal ones, whether those surplus values be derived from the unequal productive power of land, or unequal trade-skill, or any other cause which is capable of producing surplus values.

Surplus value may therefore either consist of the consumer's unbought surplus, as when he gets a thing at a price less than that at which he values it; or producer's unearned surplus, which nearly coincides with rent in the sense used by Ricardo.

The first of these has been already dealt with. The second, or producer's surplus, is usually expressed graphically by saying that if S S′ is the supply curve, that is the curve of cost of supply of final increments, then M P S is the rent.

But this, as has been explained above, is a superficial view, and its inadequacy is seen by considering a case in which S is of the descending order. What, we may

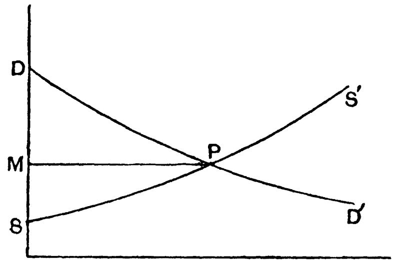


FIG. 33.

ask, is the rent here? Is it —PSM? in other words, does the manufacturer receive a bonus from some one representing the landlord upon the goods which cost him most to make? This is felt to be obviously absurd.

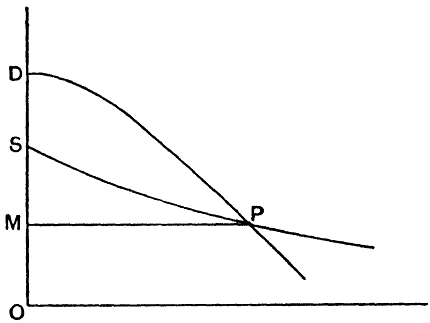


FIG. 34.

It is well known that at a good coal mine increased outlay on machinery and efficacy of production cheapens cost of production as the output of coal is increased. A mine at which coal cost 10*s.* a ton to put out when only 300 tons a day are raised, will

perhaps be enabled to put out coal at 8*s.* when the daily output becomes 800 tons; and when the daily output increases perhaps to 1,500 tons, the cost will be still more diminished. Here then the supply curve is clearly of the descending order. I will give a concrete instance of this. In a recently opened mine in Yorkshire the following computation has been made for me by the Manager, himself one of the most competent

Tons per week.	Cost per ton.	
	s.	*d.*
2 746	8	3
4 323	7	6
5 069	7	2
5 456	7	0
5 653	6	9
6 159	6	4
7 015	6	3
8 000	6	2
9 000	6	1
10 000	6	0

mining engineers in the country. The second column shows the cost per ton of raising the coal (including wages, timber, sundry stores, rates and taxes, management, colliery consumption, and renewal of plant), when the quantities are raised per week which are given in the first column. It also includes a payment for royalty to the landowner, which is a uniform sum per ton raised.

On a diagram we get the annexed curve, s s′, Fig. 35.

The demand curve of coal is not known, but at the colliery in question the average price obtained at the pit's mouth is about 6*s.* 6*d.*, and it is found that to

produce less coal a week than about 6,000 tons is to work at a loss. Here, then, the demand curve cuts the supply curve negatively. The tendency is therefore at this mine to go on and produce as much as possible, for the seams being good increased production does not mean increased difficulty. Hence the output is rapidly increasing, being limited only by the amount they can get out of the shaft. They are now proposing to use increased hauling machinery, and have put in electric

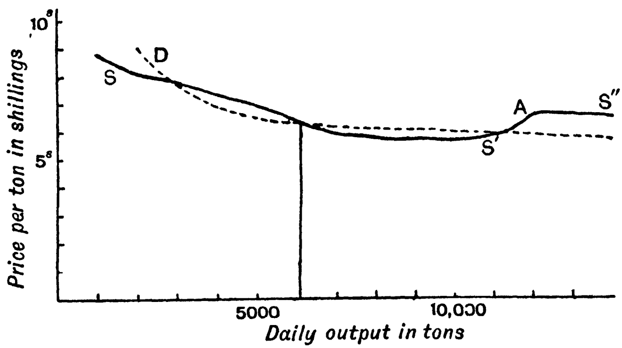


Fig. 35.

coal-cutting machines. A point at last will be reached (say at a weekly output of 11,000 tons?) when human ingenuity will be exhausted, and whatever amount of coal can be got out below, it cannot be hauled up out of the pit unless at a speed of working that would be too dangerous. Then the question will arise of sinking a second shaft; but this would produce a hump on the supply curve as at A, and the demand curve shown by the dotted line might cut the hump, and, by producing a point of stable equilibrium, put an end to further output. I believe that this mine, like many others in

the country, chronically works with the demand curve cutting the supply curve negatively, or in a state of unstable equilibrium. Hence the oscillations of colliery shares.

But is there no rent in that mine? I mean not merely in the sense of money paid for leave to dig and take away the coal, but is there no true surplus in the sense that one part of the mine yields more coal to a given expenditure of capital and labour than another? Certainly there is such a surplus. In every mine such surplus is recognized even in wage-paying, and differences of price per ton to the hewers who extract it are regularly fixed for different working-places.

In this particular mine the differences are not very considerable, except in certain parts; for the seam is of particularly rich and uniform quality. But there are differences, and these differences are in most cases dealt with by putting the steadiest and most regular men to work in the best places. As a result their pay-sheet, even for the same amount of labour, shows more coal gotten and higher pay. This higher pay is distinctly of the nature of rent. In other cases where exceptional difficulties exist, the coal is hardly worth getting; the labour is heavy, and an extra tonnage rate of pay has to be given to the men.

In other parts of the mine, where very exceptional facilities exist, a lower pay rate per ton is given. The difference between this and the higher pay rate given in less favourable parts goes into the owner's pocket. It is clearly of the nature of Ricardian 'rent.'

If, then, this surplus or 'rent' exists, what curve represents it?

Clearly it cannot be shown by the supply curve, for it is a positive not a negative quantity.

It cannot be treated as a short-period or temporary phenomenon; it is a permanent one, remaining during the whole life of the mine.

Suppose that S in the figure represents the supply curve. Then if the output were only 100 tons a day, the cost per ton would be (say) 13*s.* 2*d.*; if 200, 12*s.* 6*d.*; if 300, 12*s.* 3*d.*, and so on; and when 500 tons were

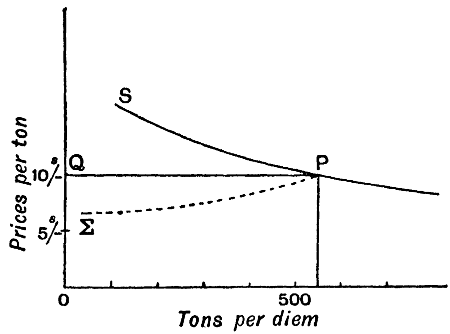


FIG. 36.

extracted daily, the price would perhaps be 10*s.* Where is the rent? It is clear that 10*s.* must represent the price per ton of raising the coal in the least remunerative part of the mine; the cost of raising the coal in the most remunerative part may be perhaps 8*s.* How can you show this fact except by drawing a dotted curve from P back to Σ on a downward slope towards the left, in such a manner that at any point it shows the surplus caused by reason of the reduction of cost of production?

And in this case ΣPQ is the 'rent,' which, as the

royalty is a fixed charge, is entirely received by the company or the men working the mine. I do not think it possible, in the face of this example, to deny that the curves of final utilities as at present usually shown require to be supplemented by surplus value curves in order to obtain consumer's and producer's surplus value.

This surplus value, as has often been pointed out, may have originated in the labour of the person who owns the mine, or his good fortune in possessing natural advantages, and its amount may be increased by changes in the demand curve with which he has nothing to do.

It is proposed in the case of land to lay fresh taxes upon it, and the ratepayers of large towns, most of whom are tenants, naturally view with lively satisfaction the prospect of having conferred upon them the power of levying taxes upon their landlords and spending the proceeds for the common benefit.

CHAPTER VII

TAXATION

THE effect of taxation upon the supply and demand curves has been so admirably dealt with by Professor Marshall in his privately printed paper on Domestic Values in 1878, and in his classical work on Economics, that I do not propose to enlarge upon it.

Briefly put, it appears that a tax increases the cost of production either by a fixed or a *pro rata* amount. Its effects upon price and output are easily seen from the diagrams on the next page.

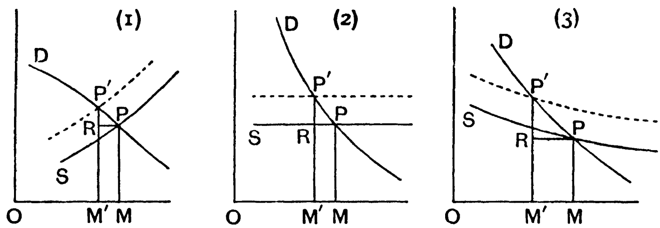
In each case the dotted line represents the cost of production after the tax has been added to it.

In case (1) of an article of the agricultural order, the price is increased more than the tax. In case (2) of an industrial order, the price is increased by the amount of the tax. In case (3) of a manufactured article, the price is increased by more than the amount of the tax.

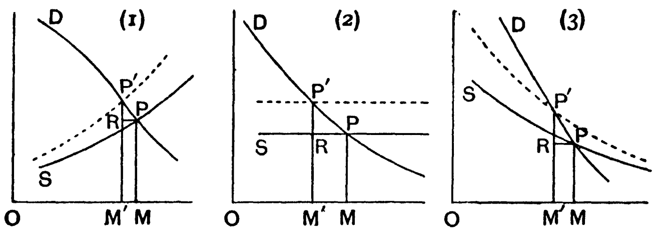
Taxes on surplus values, such as rent, obviously have no effect on prices, for the very reason that they are only levied on surpluses and thus do not increase the cost of production of the final increment. A tax on tithes would not increase prices, for it would come out of the parson's pocket, and not the farmer's. But, of course, if tithes were remitted, production

would be cheapened, and, *pro tanto*, prices ought to fall.

It is to be observed that in computing the amount of an *ad valorem* tax, if care is not taken a misconception may arise. Take, for instance, the rating of a house. It is sometimes said by extreme Socialists that the



Fixed Tax.



Ad Valorem Tax.

FIG. 37.

rates ought to be 20*s.* in the £, meaning that nothing should be left for the landlord. But the rates are levied on the letting value of a house, the tenant paying the rates, and this letting value is assessed from time to time. Therefore a house that would be worth £100 a year's rental if no taxation existed, would if a tax of 20*s.* in the £ were exacted become worth £50 rental, the tenant paying £50 to his landlord, and £50 to the

local authority; that is, £1 on each £1 of the letting value, which would of course sink to half its former value, and have to be re-assessed accordingly. Thus, a rate of 20*s.* in the £ only absorbs one-half the rental. And so in proportion a rate of 6*s.* 8*d.* in the £ absorbs not one-third but one-fourth of the rental. And generally if x is the rate in shillings in the £ the proportion of rent absorbed by the rate will be $\frac{x}{x+20}$.

CHAPTER VIII

THE COMPOSITION OF SUPPLY AND DEMAND CURVES

THE demand and supply curves may be compounded. Thus, for instance, the demand curves may be added together, and the resulting demand curve will be the combination:

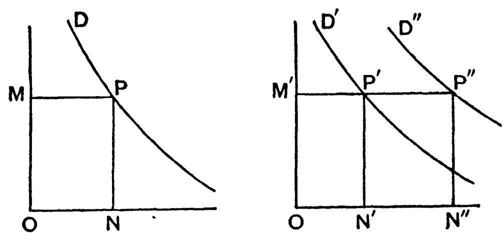


FIG. 38.

Thus, if D and D′ be two demand curves, we need only add the abscissa ON of D to ON′ of D′ in order to get the resulting curve D″, where $ON'' = ON + ON'$.

Supply curves of the ascending order can also be similarly compounded. When, however, they are of the descending order they cannot be so easily united. For supposing we had two supply curves of the manufacturing order, S and S′, and assume that they were exactly similar. Then at a price OP the amount made would be MM′, one-half by each manufacturer. But the equilibrium would be quite unstable. Unless they agreed to divide their output in certain proportions it would end by one completely underselling the other,

and one manufacturer might probably make N N′ and the other nothing at all. In fact, however, the supply curves cannot go on descending indefinitely. Even with the largest company there comes a period when they can make no more, and then competitors get their chance. But it is to be observed that when the competitor S′ abovementioned has got his supply up to N N′, his defeated rival S can do nothing to oust him by merely making a little. For till he begins to

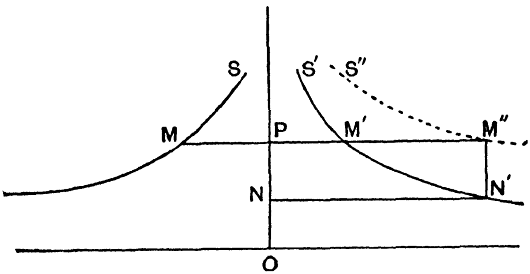


FIG. 39.

make on a big scale he cannot undersell S′. His only plan is to raise money, and make an effort to try and oust S completely, which if he does then S′ will be ruined and S take his place. It is like a game of beggar-my-neighbour, in which the winner adds to his pack all his adversary's lost cards.

These considerations show how different competition is in agricultural production from competition in manufacturing or industrial production. For each farmer who tries to increase his output is so to speak going up hill. Each increased amount he produces is at a greater and greater relative cost, and circumstances automatically put a drag on his efforts to undersell and

oust his rivals. And, on the other hand, each restriction put by a farmer on his output lessens his losses and risks. The less he makes the nearer he fights, so to speak, to his base. Therefore farmers do not dread one another much as rivals; the success of one but rarely hurts another.

But when we turn to industries, a different rule prevails. Then each success gained by a producer puts him in a more favourable position to undersell his competitors; each victory adds to his own strength by cheapening his cost of production and increasing that of his rivals. Hence the big men drive out of the market the small men, until by the formation of trusts the big men are themselves driven out by syndicates. And this I think is the meaning and reason of the want of cohesion among the manufacturers in many trades. Each fears the other. Instead of a field of easy-going farmers we have a scene of fierce competition—competition often too exciting for the competitors. It is a terrible feeling to be carrying on a business with the knowledge that if your neighbour can once get his production expenses down a penny below yours he may not merely take away a customer or two, but in a short time ruin you outright.

This feeling is the basis of trade combinations, which, originally formed to relieve the tension of over-competition, soon develop into monopoly associations which devour their neighbours and prey on the unlucky consumer. The effect of these observations will be seen when I come to treat of foreign trading, and of the cataclysms which may be produced by changes in the modes of production, changes accentuated by the extraordinary progress of scientific discovery.

CHAPTER IX

MONOPOLY

When a producer is in such a position that he can restrict the supply of a commodity, he is said to be a monopolist. His power as a monopolist may be partial or total. If he were in the position of sole possible producer his monopoly would be absolute, as in the case of a man who possessed the Cumberland graphite mines years ago. A patentee of an invention was once an absolute monopolist. Now in theory he is so no longer, for there are means of compelling him to permit his invention to be used at a price fixed by arbitration.

Partial monopoly is caused whenever impediments are put in the way of competition. The formation of rings and syndicates restricting sales under certain prices operate in the direction of creating partial monopolies. So also may raising of railway rates, or the imposition of import duties on foreign goods. The action of Trades Unions is directed to the formation of partial monopolies, sometimes by arrangements between workmen, sometimes by 'coercion,' sometimes by 'moral suasion,' sometimes (as has happened in the glass trade) by limitation of the number of apprentices.

These Trades-Union monopolies are usually partial only. For this reason every possible discouragement is given by men of one trade to the learning of that trade by men of another trade. For though, of course, if there were a universal workmen's combination a knowledge of divers trades would enable men easily to

denude any one trade of its men, and throw the direction of its labour into another, and thus starve that trade into submission ; yet, as things are, men in one trade are far too afraid that during a strike the men of a kindred trade will act as rivals and fill their places.

A monopoly value for any particular article is shown on a curve diagram by making the area M R W T a maximum. Where S P S′ is the supply curve of final cost of production, and W T is the successive intermediate-cost curve corresponding to the manufacture

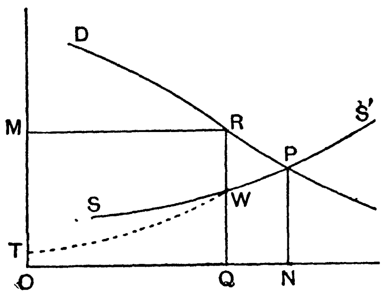


FIG. 40.

of an amount O Q, if S S′ were a horizontal straight line, as in industrial supply, and the intermediate cost-of-production curves coincided with it, as will commonly be the case in industrial cases, then the point R would be that at which a rectangular hyperbola with S Y and S S′ as axes touched the demand curve. (For it must be remembered that though a demand curve generally cuts any hyperbola positively, and would therefore rarely, if ever, touch a rectangular hyperbola drawn with O X, O Y as axes, it would of course almost always touch somewhere a properly chosen rectangular hyperbola with S Y and S S′ as axes). And in that case

the rectangle N S W R would be a maximum, and hence the excess of N R M O, the total price obtained, over S W M O, the total cost of production, would be a maximum, and if P R S is a tangent at R, then P R = R S.

The use of the hyperbola in problems of monopoly is due, I believe, to Professor Alfred Marshall.

Monopolies may be created also by restricting the supply of labour. This is the means which Trades Unions commonly employ. The high rate of wages which has been maintained in the building trades and the coal-

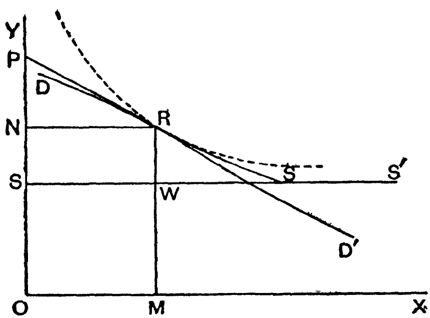


FIG. 41.

mining industry is no doubt partly due to the fact that foreign competition is impossible, so that those in the trade possess to some extent a local monopoly. But other monopolies of labour to be successful must as a rule be partial. A universal monopoly of labour would be almost a contradiction, for monopoly involves restriction. Some plan by which labour was artificially restricted, the unemployed labourers who were not allowed by Union rules to work, being thrown on the rates, would be apparently the widest form which a labour monopoly could take.

CHAPTER X

INTERNATIONAL TRADE

HITHERTO we have dealt only with a producer and a consumer. We must now consider the case of two countries (or persons), each of whom is using and also producing the same or different articles. This is the problem of International Trade. Suppose there are two countries which have no trade relations, either

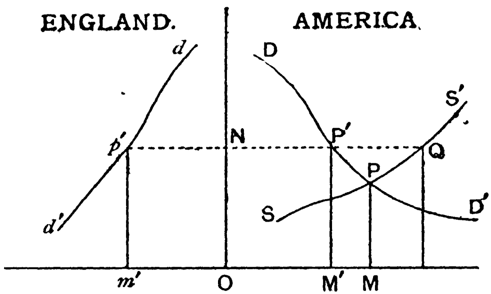


FIG. 42.

by reason of war or for some other cause, and that we can draw the domestic supply and demand curves of each of them with respect to some commodity, and hence determine the price and amount of home-consumption in each. Now suddenly let all barriers be removed, and, for the present, let cost of carriage be neglected. If the price of the article is identical in each country, no trade will take place unless the article

is of the manufactured order, when possibly the intercourse will result in the whole manufacture being transferred to the nation that could make it cheapest if the amount made were largely increased.

First, however, let us suppose that the cost of production of the article is of the ascending order. If the price in each country is the same (which could only happen by a miracle), then no disturbance of trade will take place. For the purpose of distinction let us suppose that the countries are America and England, and the

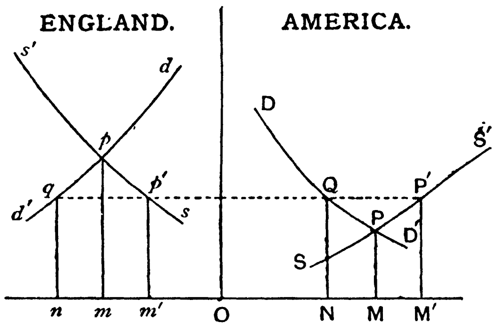


FIG. 43.

article supplied is wheat. Suppose, in the first place, that England grew no wheat at all. If we put the curve diagrams of England and America back to back, as in the figure, it will be seen at once that if P M be the price and O M the output in America previously to the trading, then when the intertrading begins the price will rise, and this will go on until the price in both countries is N O = P′ M′, the point P′ being such that America's total output will equal N Q, of which she will export P′ Q, equal to p' N the amount imported into England. America will then supply the united demand

of the two countries. If England also produces wheat, but at a cost greater than that in America, then the effect of intertrade will be shown by Fig. 43. The price, which in England was pm and in America PM, will now become equal in the two countries, and will be $P'M' = p'm'$, the points P' and p' being taken such that $P'Q$, the amount exported from America, is equal to the amount $p'q$ imported into England. The consumption in each country will be ON and on re-

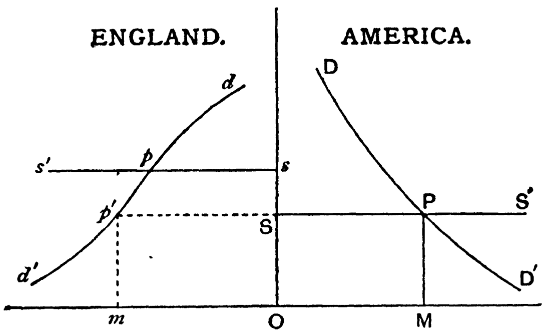


FIG. 44.

spectively, the production in the two countries will be OM' in America and Om' in England.

Now let us suppose that the article is of the industrial character, and that America, as before, has a uniform advantage over England in the making of the article. The effect will obviously be to transfer the whole manufacture to America, and the price in both countries will fall to PM, the total amount made in America becoming mM.

Let us now consider the case of curves of the manufacturing or descending order. Here the problem becomes more complicated, and instead of a slow change

we may see the removal of trade-barriers accompanied by changes of price of a very large description.

The sudden removal of barriers may cause the goods of one country to invade another like a flood, ruining the trades that exist there; as when, for instance, in Turkey after the Crimean war machine-made cotton goods were suddenly introduced into a country where all weaving was previously done by hand.

It is impossible to examine all the cases. One or two samples must suffice.

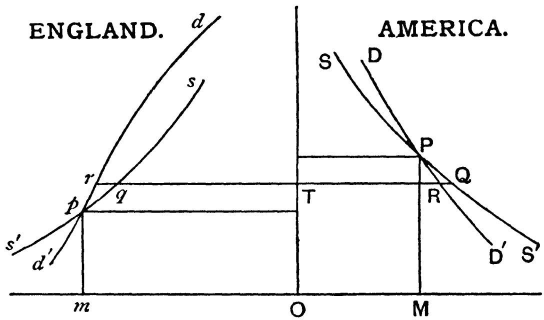


FIG. 45.

Suppose the supply and demand curves were as in Fig. 45, the prices in the two countries being P M and *p m* respectively. When trade commenced, English goods would at once be offered in the American market, the price would rise in England, and consequently the English demand be restricted.

The goods thus thrown on the American market would lower the price there, and therefore increase the amount demanded. The increase of amount demanded would induce American manufacturers to compete with English manufacturers. The Americans would, as the shape

of the curves show, soon produce more than they wanted, until R Q exported from America $= r\,q$ imported into England, and the price would rise in England and fall in America till it stood at O T. Any attempt on the part of American manufacturers to exceed their output of T Q would involve more being thrown on to the combined markets of England and America than the demand of those countries could carry at the price at which the goods could be offered; in other words, Q q, the total

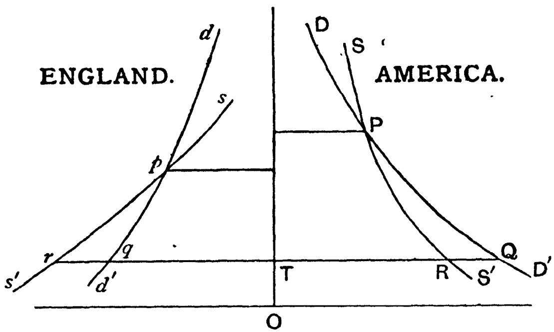


FIG. 46.

production of both countries, would exceed the total demand R r, at the price, O T, at which the article could be offered, so that in the end the position of T would be a point of equilibrium, at which, at the price O T, the combined demand of the two countries would equal the combined supply. In certain states of the curves T would not be a point of equilibrium; all would depend on whether the lowering of price caused the supply at that price to exceed the demand. Then further manufacturing would stop. There might be other points of equilibrium, and ultimately it would be probable that

the whole of the supply would be made in one or other country.

Again, if Fig. 46 represented two pairs of curves, of which the one on the right-hand side was in unstable equilibrium, then trade between the countries might result in a fall of price to a lower point than previously existed in either country, namely to O T, in which $QR = qr$ (the amount exported from England to America), and stability would only be attained when such a point had been reached that further production by England would so glut American demand that the goods could not be sold at a profit. In other words, that Rr exceeded Qq.

But in all cases of supply curves of the descending order equilibrium is very uncertain. *Theoretically*, the tendency is for the whole production to be in the hands of the person or country that can produce cheapest. It is only the sheer fatigue of making money, the death and retirement of leaders of industry, the difficulty of creating a staff of workmen, inventions, and changes of wages and industrial skill and conditions, that prevents this theoretical effect taking place. Two manufacturing rivals are like men pulling against one another on a rope, one on each side of the summit of a hill. When one of them is once pulled over the summit the other can run away with him. But if the men were pulling on each side of a hollow, as in rival production of the agricultural order, then when one got pulled down a little his opponent's task would become harder; so they would come at last, like a marble in a bowl, to a position of equilibrium.

It is no part of my purpose to show the effects of these laws upon the social and economic conditions of

communities, but it is evident that they might be made the subject of very interesting speculations.

Primitive societies are mostly agricultural and industrial. The curves being thus of the ascending order, rivalries are made difficult, and the sharpness of competition is lessened. But with the advent of manufacturing industry, profits and wages, which had previously been in stable equilibrium, now are governed by descending supply curves and with increasing frequency become unstable, thus throwing a great strain on the constitution of the country, and disturbing the social condition of its inhabitants.

Endless combinations of curves may be drawn. Only in curves of the ascending order will stability of conditions be generally found. In the rest there is a liability for every disturbance to run to extremes. The trade is like a globule of mercury on a plate. The slightest inclination causes it to rush from one position of rest to another, sometimes in a most distant position. It is true that the ultimate position of rest is for the time stable, but the path from one stable position to another instead of being gradual is spasmodic.

Various causes prevent the full effect of the instability of manufacturing prices being felt. The necessity of carriage from one place to another is an obstacle. The impossibility of suddenly creating the necessary skilled labour is another; timidity in enterprise, and the persistence of old habits is another; and of course protective tariffs would also help. The intervention of a sort of arbitration board, to retard sudden transformations, would possibly have some effect, if it were not probable that the problem is so complicated that the medicine might produce worse effects than the disease.

We have now to consider the effects of an import duty. Obviously it will add to the price in the country of consumption, but not necessarily or usually to the whole amount of the tax. In the case of articles of the agricultural order, where the supply curve is ascending, the tax will be shared between the exporting and importing countries. The effect of an import tax, or an export tax, or of an increase of freight, is exactly the same so far as its effect on prices is concerned. This is shown by Fig. 47. If the line Pp' shows the

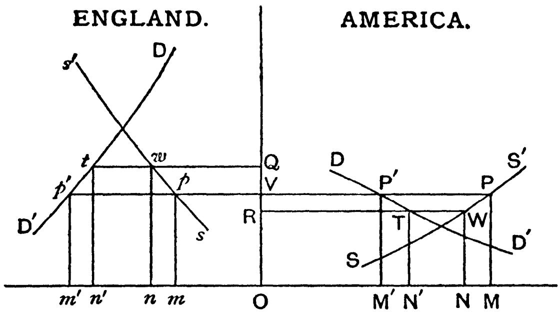


FIG. 47.

price line when trade is free, then if an import tax = Q R is put on in England the price line will be disturbed. The price in America (the exporting country) will be lowered, the price in England (the importing country) will be raised. The quantity grown in America will be O N, of which she will consume O N′ and export N′N. The quantity grown in England will be O n, of which she will consume the whole as well as $n\,n'$ which she will import, and N N′ will be equal to $n\,n'$.

The ratio of Q V to V R, that is, the ratio of the rise in price in England to the fall in price in America, is

generally considered as the indication of the question 'Who pays the tax?' It will at once be seen that this ratio depends on the rapidity of the change of magnitude of P P′, the amount exported, as compared with the vertical distance through which P P′ has to move in order that in its new position, T W, it may still be equal to *t w*. This depends simply on whether the curves in America or England are steep or sloping. The country in which the demand curve and supply curve are the most easy, that is, least inclined to the axis of x—that is to say, in which demand is least keen and increased supply most easy to effect—will in general have the best of the changed circumstances, and be least affected by the tax, which in common language will be said to be 'mostly paid by the other;' and this will especially be the case in articles of the manufacturing order.

On the other hand, in countries where the demand is very keen, and where a difference of price causes but little difference in the amount of the article consumed, a considerable change of price, measured in a vertical direction, will be required in order to effect a given alteration in the amount imported, and in such cases the consuming country will generally pay most of the tax.

As we have seen, the demand of a country for foreign produce, or its export to another country of its own produce, begins after its own supply or demand at any given price has been exhausted or satiated.

Thus, for instance, suppose that the curve Fig. 48 represented the English curves for wheat. The price would be P M. But if a foreign country came in, and offered wheat in abundance at a price O N′, the price would obviously sink to that amount, the home-grown

supply would go down from O M to O M′, and the quantity M′M″ would be imported, so as to make up a total consumption of O M″ at a price M″P″. The home-grown supply, O M′, would of course first be consumed, the foreign import would supply the remainder.

Again, suppose that the curves, Fig. 49, represented the American curves for wheat. Then *p m* would be the

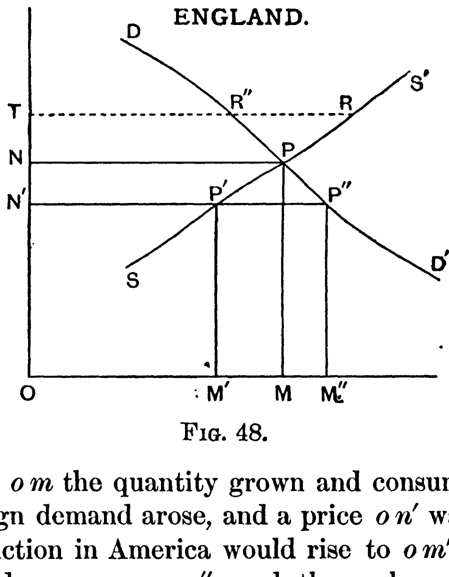


Fig. 48.

price and *o m* the quantity grown and consumed. But if a foreign demand arose, and a price *o n′* was offered, the production in America would rise to *o m′*, of which she would consume *o m″*, and then when her home demand at the price *o n′* was satiated she would export *m″ m′*, the balance of her production.

If in the above figures other horizontal lines were drawn, at various distances from O M and *o m*, then it is clear that the parts intercepted between the demand curve and the supply curve would represent either

exports or imports. Thus, on Fig. 48 the intercepted part, R′R″, of the dotted line would represent an exported amount, for it would indicate that at the price O T there was an excess of supply over demand. In Fig. 49, the intercepted part of the dotted line would represent an import, for it would indicate an excess of demand over supply.

It is clear, therefore, that in either of these curves the intercepted portions corresponding to various prices

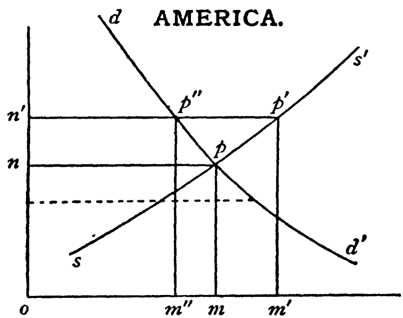
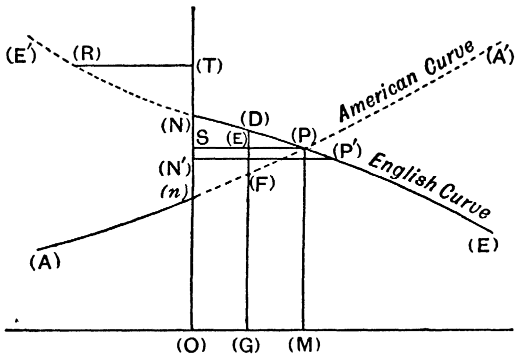


FIG. 49.

represent the dealings which the country is prepared to have with the foreigner. They are surplus demands, or surplus supplies, and taken in a group they represent the desire for foreign trade, and a curve may be constructed out of them.

This may be done by drawing a new curve, in which the ordinates shall be prices, but the abscissae shall represent the excess of the supply over the demand in a country at those prices, or the excess of the demand over the supply; that is to say, the intercepted portions above described.

Thus, for instance, if we consider Fig. 48 as being a supply and demand curve applicable to the supply and demand of wheat in England, we may (Fig. 50) construct a curve (E′E) in which an ordinate (O N′) should have an abscissa (N′P′) equal to P″P′ on Fig. 48. This would indicate that at a price (ON′), (N′P′) (which has been made equal to P″P′ in Fig. 48) would be imported. If the price rose to (O N) nothing would be imported



Exports are shown by a dotted line, imports by a continuous line. The ordinates represent prices, the abscissae represent quantities.

FIG. 50.

or exported. If the price rose to (O T) then (T R) (=R′R″ in Fig. 48) would be exported.

Here then we should have a curve of foreign trade. The curve would represent at various prices how much the country (England) was prepared to take, or to give. The branch N P E′, shown in a continuous line, would represent imports; the branch N R E′, shown by a dotted line, would represent exports. The curve would cut the axis of Y in a point N corresponding to N on Fig. 48, and would pass to the other side of the axis, representing

excess of English supply over English demand. The whole curve would represent the condition of England as a consumer of wheat from abroad, or a producer of wheat for exportation. In the same way, another curve can be constructed for America from the curves in Fig. 49. Here the distance (O N), Fig. 50, is equal to *o n*, Fig. 49, and the curve has also two branches, an exportation branch, shown by a dotted line, and an importation branch, shown by a continuous line. We will draw this curve so that its dotted branch is on the reverse side to the dotted branch of the English curve; in other words, put the curves back to back. When this is done, their point of intersection (P) will show the price (P M) that would prevail in both countries if trade were free and freights were neglected, and (O M) will be the amount that would be exported from America into England.

These curves, E E′ and A A′, would not be value-in-use curves, nor cost-of-production curves, but a compound of each. Each would have two aspects. Each would represent prices corresponding to an excess or a deficiency of supply in one of the countries. We may call them a pair of exchange curves. They are in every way symmetrical about the vertical axis, and about the horizontal axis. They are, however, not governed by the general laws that govern the usual supply and demand curves, but being symmetrical, and each being both a demand and also a supply curve, the application of the laws of supply and those of demand must be applied to the exchange curves in proper character, by considering each change in them as a result of some change in the supply and demand curves, or either of them, out of which it is composed.

The effect of a tax of a given amount would be shown by moving the line (PM) to a new position (DG), such that DF would represent the amount of the tax; (FG) being the price of wheat in America, (GD) being the price of wheat in England. And the ratio in which the point (E) divided the line (DF) would represent the ratios in which the tax was borne by the respective countries. It will at once be seen, therefore, that the proportion in which the tax will be divided between exporter and importer will depend on the inclination of the respective demand curves. If one of them be very flat, then the fact of a tax and the readjustment of price caused by it will be small. If one of them be steep, then the country whose curve it is will experience a considerable change of price by the duty.

Curves such as that shown in Fig. 50 are, of course, mere derivatives from such curves as are shown in Figs. 42 to 47, which I think are the clearest for general use.

These lastly mentioned curves, Fig. 50, as will subsequently be seen, correspond to Professor Marshall's foreign exchange curves, into which by a simple geometrical construction they can be easily transformed.

Although they may be employed to represent a series of surplus productions and excess demands, they do not appear to me sufficient fully to represent the conditions of the export and import of two articles balanced one against another, or even of a single article as balanced against money.

In the previously proposed diagrams it has been shown that properly to exhibit the problem presented by the import and export of an article which is made

and consumed in each of two countries you cannot do with less than four curves. For unless there are differences in the supply and demand curves of the two nations no exchange would take place at all. The costs of production and values in use in the two countries are therefore the four elements of the problem, and the best form of solution is that which brings them all in, so as to show the part which each is playing in the solution.

As has been said, freight may be regarded as equivalent to a natural import duty. In fact, an export duty, an import duty, and a freight are all one so far as their effect on production and price is concerned. Each simply operates as if the distance was increased between countries; and to put an import tax on American commodities is (apart from the gain to the revenue) as if we insisted that vessels from New York should come to England by way of Cape Horn and the Cape of Good Hope.

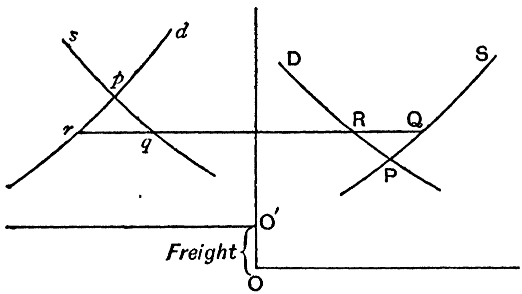


FIG. 51.

The effect of freights may be geometrically shown by shifting the axis of x upwards through a distance O O′

equal to the freight. The tax and price questions may then be solved as before. The distribution of the payment of freight between producer and consumer will follow the same rule as the distribution of the payment of a tax.

From the figures given it is evident that a tax, like a handicap, to be prohibitive must not be less than the whole difference of the productive power of the respective countries. If one boy can run a mile in a minute less than another, no purpose will be served by giving the second boy less than a whole minute's start; and therefore import taxes to be effective must be pretty heavy. Of course, limitation of imports involves the limitation of exports. This result is recognized by the Americans, who frankly say they do not want any foreign trade at all. Of course, however, a nation that desires large exports must be prepared to admit large imports. One reason why America gets such a low price for her wheat possibly is that she refuses to take payment in the goods that her neighbours can most easily offer, and thus forces them to incur expense in finding something to pay her in.

The economic effect of bounties is of course the reverse of that of taxes, and is easily shown on the curves.

Dumping is usually the result of the imposition of a protective tariff; it may be defined as the exporting of goods for a price less than that at which they are sold at home. It is generally the result of a scheme of production of the manufacturing or descending order, whereby more is made than will probably be wanted by the home market, and the remainder sold off in foreign markets for what it will fetch. Of course it

benefits the foreign consumer, and to some extent the foreign trades that can use up the dumped article. As a rule it is a characteristic of a trade that is in an uncertain condition, for no manufacturer will go on selling below cost price if he can help it. Thus when the American bicycle manufacture commenced, manufacturers, in expectation of a greatly increased American demand, put down plant and factories larger than the existing American demand warranted. In order to keep the plant going they made more than was wanted in America, and having sold what they could in America, dumped the remainder in England, completely upsetting for a time our home bicycle trade. But this did not last, nor was it intended to; when the anticipated American demand increased it proved capable of taking up the whole of the American manufacture, and the dumped American machines disappeared from our markets. England, as it was said, 'regained the bicycle trade.' It appears also worthy of note that this American dumping was not done with any sinister views on our bicycle manufactures, but solely as part of the preparation for a largely increased use of bicycles in America. The plan of trying to ruin a competitor by persistently underselling him at a loss rarely succeeds, except against a bankrupt rival; a healthy and solvent rival usually buys the dumped article and finishes it, and perhaps re-exports it with complacency, and hopes that dumping may continue. Sudden and unexpected dumping is of course apt to unhinge businesses that are not on a very firm footing. If a very low import tax were placed on dumped articles it would probably be paid by the exporter.

These curves show to some extent the effect of protective duties.

The results appear to give rise to the following arguments for protection:—

1. That very sudden and rapid changes of price caused by the action of neighbouring nations unhinge trade, and damage a country's powers of production; and that it is wise to have some protection against such cases.
2. That protective tariffs may prevent the emigration of home capital to protected countries, whence from within the protected zone the producer might supply both his own and the foreign country at his choice.
3. That since it is admitted that the fiscal action of other countries can injure us, protective tariffs enable us to force them, on some occasions at least, to abstain from that action.

The arguments on the other side appear to be:—

1. That although protective tariffs may lessen imports, this very diminution of imports must also cause a diminution of exports.
2. That the power to purchase goods cheaply always in the long run gives a country more commercial and industrial advantages than would be obtained by protection.
3. That it must always ultimately benefit a nation by a process of elimination to have its capital directed to those industries for which it is or may become most fit; and that protection must always, *pro tanto*, tend to prevent progress.

It is no part of my duty to weigh these arguments one against another. Perhaps the real truth is, that

protection is a medicine, and that before giving it you ought first to find out whether the patient is ill; next, whether the proposed drug will make him better; and thirdly, how much you are going to administer. The course you adopt will probably depend more on empirical therapeutics than on theoretical considerations.

Diagrams of course cannot decide the question of free trade as against protection. They can only display the conditions of the problem.

CHAPTER XI

TRANSFORMATION OF CURVES. MARSHALL'S CURVES

It has perhaps occurred to the reader during the perusal of the foregoing pages to ask what is the character of the curves that have been considered. Suppose, for instance, that we had a pair of curves—a supply curve, and a demand curve—of cloth in terms of linen. Are we to regard one of the curves as relating to the producer of linen, and the other to the producer of cloth? or are we to regard one of the curves as representing the utilities to all persons of linen as compared with the utilities of cloth, and the other as representing the relative cost of production of one in terms of the other?

In the first of these senses, one of them would be a demand-of-cloth-and-supply-of-linen curve, the other a demand-of-linen-and-supply-of-cloth curve. In the other sense one curve would purely relate to relative utility, the other curve would purely relate to relative cost of production. The latter is obviously the sense in which the curves have been used hitherto, with the exception of the derivative curve represented in Fig. 50. For in one curve we have compared the relative utility of cloth and linen to consumers in general, without any reference to their expenses of production; in the other we have compared their

relative cost of production, without any reference to their utility. And this is the case whether we consider the producer as also consuming part of what he produces, or whether the article is one which is not consumed by any one who produces it. For it has been tacitly assumed throughout all the description of the curves for any country that the curve of cost of supply of an article applies to all persons supplying it, whether or not they also consume it, and that the curve of value in use is common to every one. And even if we had estimated the article in terms of another article instead of money, the same thing would have been true. One curve would have represented the ratios of the values in use of cloth and linen, when different quantities were consumed; the other their relative cost of production when different quantities were made—each in terms of the other. It would have been assumed that the conditions of manufacture and of enjoyment of both the commodities were the same to every one.

If this assumption is not made, if we are to consider two persons or two countries with different relative capacities of production for cloth and linen, and each with different ratios of value in use, we get not two curves but four. We should have A's curve of ratios of cost of production of cloth in terms of linen, and B's curve of ratios of cost of production of linen in terms of cloth. And these curves would not be the same for the same quantities of the articles. Also we should have A's curve of ratios of the values in use to him, and B's curve of ratios of the values of the articles in use to him. And for this reason, in dealing with international trade, where the circum-

stances of two countries are being considered in which the conditions of manufacture will be different, as well as the conditions of consumption, it has been necessary to use four curves, one pair for each country.

In drawing these curves the supply and demand of only one article has been considered, and that supply and demand have always been balanced against money. For internal trade, and especially when the exchange of the article is on a small scale, this way of regarding the conditions that fix the price is preferable. The restrictions on the proportion taken of any article are due to the fact that something else is preferred, rather than to a failure of money; and the influence that the exchange has on the general rate of prices is neglected.

But when articles are compared one against another instead of against a standard like money, the problem becomes more complicated. If we are comparing the decrease of price offered (say) for linen as more and more is supplied, we treat money as an invariable standard, and simply consider how far the supply of linen satisfies the demand for it. But if instead of considering the demand for linen in terms of money we consider it as expressed in cloth, we are then involved not only in considerations of the diminishing utility of linen as it is supplied, but also of the changing utility of cloth which its supply brings about; and hence it is desirable so far as possible to consider alterations of value or cost with regard to some uniform standard of wealth in general, such as money, in order to keep the mind fixed on that article only whose market values we are considering.

It may be occasionally desirable, in dealing with international exchanges, to consider the influence of the sale on the article given as well as taken, and that the curves should be symmetrical—representing the barter of two articles rather than the purchase of one.

This wish to eliminate a money standard, and to make the curves into symmetrical curves of barter in which what is true of one curve shall always be inversely true of the other, and in which a curve that is a curve of demand from one point of view shall always be a curve of supply from the other, led Professor Marshall in 1878 to produce a new form of curve.[1]

The essence of Professor Marshall's system consists of making the ordinate and abscissa of the curve each represent total quantities exchanged, instead of taking the abscissae for quantities and the ordinates for prices per article.

In such a demand curve, the abscissa and ordinate of any point represent, not, as before, the quantity demanded and the price that would be given for each article, but the quantity demanded and the total that would be given for it. Hence then, if x and y represented the abscissa and ordinate of a point on the price curve, where x was the quantity taken and y the price per article, the abscissa and

[1] His essay (*The Pure Theory of Foreign Exchange*) was privately printed in 1878. An abridged account of his system has been given by Professor Pantaleoni in his *Pure Economics*, translated into English in 1898. It is to be greatly regretted that ill health has prevented the publication of a fuller description of these curves, which will no doubt appear when the second volume of the *Principles of Economics* is published.

ordinate on Marshall's curve would be x and $x\ y$ respectively.

Whence it follows that in a Marshall's curve if x the abscissa $= o$ the ordinate $x\,y$ will also $= o$ (unless in the extreme and impossible case in which y, the price which would be given per article, should by reason of its rarity rise to infinity). Therefore Marshall's curves always start from o, the origin.

Marshall's curves are symmetrical, in the sense that

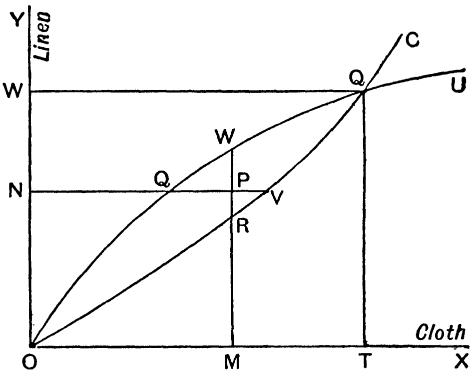


FIG. 52.

the ordinates and abscissae are homogeneous quantities, each representing totals.

Professor Marshall only uses these curves for the explanation of foreign exchanges, but of course they could be used for all purposes just like price curves, and their intersection shows the amounts exchanged. Price, instead of being represented by an ordinate, as in the ordinary form of price curve, will be represented by the ratio of the amounts exchanged, and will therefore be a ratio of the ordinate to the abscissa, that is to say the tangent of the angle which a line from

that point to the origin makes with the axes of co-ordinates.

Rents and surplus values, instead of being areas representing amounts multiplied by prices, will now be represented by lines, which however are not very easy to show except by the use of supplemental curves (see Chap. XII).

Thus, for example, in Fig. 52 if O U be the curve of relative utility of cloth and linen in any country, and O C the curve of relative cost of production, then O T cloth will be exchanged for T Q linen, and tan T O Q that is the ratio of Q T to O T, will represent the price per yard of cloth in terms of yards of linen. If less were exchanged, then for O M cloth, which cost R M linen to make, you could get M W of linen; you would therefore increase your production of cloth. On the other hand, for O N linen you would want N Q of cloth, but the maker could afford to give you N V. So the exchange would go on till Q was reached, beyond which point it would not proceed.

Each Marshall's curve can be converted into two price curves, according as we choose to make linen the price of cloth, or cloth the price of linen; but a difficulty may arise in drawing them on account of a necessity for a change in the ordinates which represent money values. For it is obvious that if an ordinate in Marshall's curve represents (say) 1,000 yards of linen (the exchange value of (say) 900 yards of cloth), and if we take an ordinate representing the price of each yard of cloth on the same scale, it will be invisible. Hence we must take some number by which to multiply the reduced ordinate, in order to get a visible diagram; and so with cloth. In order to do

this, and at the same time to convert *amounts* exchanged into prices, we shall have to take a pair of standard lines parallel to the axes, such that O R′ represents a convenient number of yards of linen, by which we must multiply the ordinates of the price curves in order to get them into manageable lengths to represent prices of single articles on a diagram; and similarly, if we wish to make cloth the price of linen, O R must be taken for multiplying the ordinates of price in terms of cloth. Starting with a given

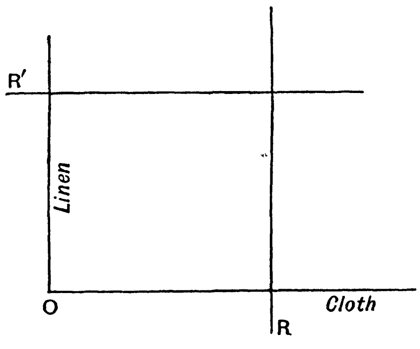


FIG. 53.

Marshall's curve, shown by the dotted line, through any point P on it we draw a line from O to meet the standard lines in n and m. A vertical line through P will meet a horizontal line through m in p, which (as will presently be shown) is a point in the price curve which represents cloth in terms of linen as a price in linen per yard of cloth; and a horizontal line through P will meet a vertical line through n in p', which will be a point on the price curve which represents linen in terms of cloth, as a price in cloth per yard of linen.

For if (Fig. 54) we put X and Y to represent the ordinates of P, so that

$$X = \text{O}\,c$$

and $$Y = \text{P}\,c;$$

and if we put $$\text{O R} = A$$

and $$\text{O R}' = B,$$

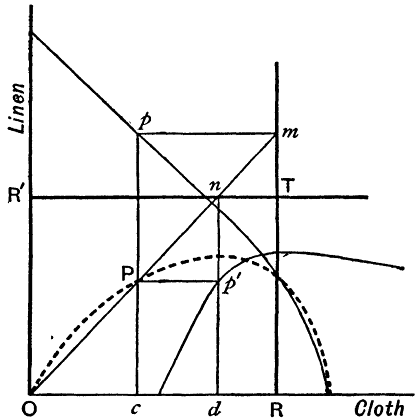


FIG. 54.

then by similar triangles

$$\text{O}\,d\ (\text{the abscissa of } p') = n\,d \cdot \frac{\text{O}\,c}{\text{P}\,c} = B \cdot \frac{X}{Y},$$

and $$p'd\ (\text{the ordinate of } p') = Y.$$

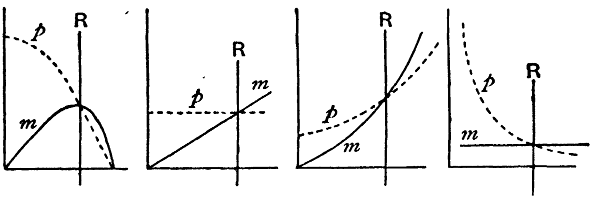
Similarly $$\text{O}\,c\ (\text{the abscissa of } p) = X,$$

and $$p\,c\ (\text{the ordinate of } p) = m\,\text{R} = \text{O R}\,\frac{\text{P}\,c}{\text{O}\,c} = A \cdot \frac{Y}{X}.$$

Hence then the curve p has an abscissa representing X, the amount taken, and an ordinate representing $\frac{Y}{X}$ the ratios exchanged, multiplied by a suitable con-

stant A; therefore the ordinate pc represents the price. It is hence a price curve of cloth in terms of linen, and when O c of cloth is taken the price per yard will be $p\,c$ yards of linen. The other curve p' reciprocally is a price curve of linen in terms of cloth.

Hence p and p' are the price-curves, which can both be constructed from the Marshall's curve, or one of them be constructed by a proper scaffolding from the other, or Marshall's curve be constructed from either. The ratio of Y to X, that is, the tangent of the angle



In these diagrams, Marshall's curves are shown by continuous lines. The corresponding curve, where ordinates show prices, is shown dotted. R is the standard multiplier.

FIG. 55.

P O c, is the price of a unit of cloth in terms of linen; the cotangent of P O C is the price of a unit of linen in terms of cloth.

The forms that some of these transformed curves take are shown in the adjoining Fig. 55. An ordinary price demand curve, p, becomes in Marshall's form a curve m, starting from O and curving round to meet the axis of X. When p is horizontal, m becomes an inclined line through the origin; a supply curve of the agricultural form, becomes a rising line; when p is a rectangular hyperbola, m becomes a line parallel to the axis of X.

The laws that govern demand and supply curves of barter do not necessarily follow the same laws that govern curves representative of price. The more boots you buy for money the less price you will give per pair; but if you are exchanging them for medals of your own manufacture the decrease of the cost of the medals, due to the increase of your output, may go on faster than the decrease of the value to you of the boots. So your price demand curve of boots in terms of medals might actually ascend, though as compared with articles in general the utility of boots would diminish in proportion as you obtained them.

There is apparently only one law that governs these curves where neither of the articles compared is money or some similar representative of wealth in general.

No one will at any one time and place offer more for a less quantity of an article than he would offer for a greater quantity of the same article.

Where one of the articles exchanged is money, then the supply and demand curves in Marshall's form of course follow laws analogous to those that govern ordinary supply and demand curves of price.

So far these curves are exactly analogous to the price curves, and though symmetrical are not truly homogeneous. For one of them represents relative values in use, the other represents relative costs of production.

There is, however, another meaning that can be given to these curves which will have the effect of making them both completely symmetrical and homogeneous with respect to the axes, so that the laws which govern one will be also applicable to the other. It is to make

them into international exchange curves, and this is Prof. Marshall's object and achievement in the privately printed essay on foreign values. He does this by no longer treating one as representing relative values in use and the other as relative costs of production; but he attaches one curve definitely to each country, making one (say) the English demand curve for German linen in exchange for English cloth, and the other the German demand curve for English cloth in exchange for German linen.

He reckons the whole of English exports of goods of all kinds in terms of and as represented by cloth, and the whole of the German counter-imports in terms of and as represented by linen, thus obtaining some of the advantages of an invariable standard of comparison. The exchange rate is then determined by the point Q where the curves intersect Fig. 52.

In the application of Marshall's curves to foreign exchanges there are, however, I think some difficulties and disadvantages. True it is that they are symmetrical, and that each article is compared with the other so as to embrace all changes in value that a supply of one will produce upon the other.

But they do not enable the whole conditions of the problem to be stated. They simply compare a German demand curve of cloth with an English demand curve of linen; but they do not show the differences produced by a change in relative utility of cloth and linen in Germany, nor a similar change in England. It is an attempt to represent a problem that really involves four curves by two only.

For Marshall's curves are really derivative curves, and according to my view they are exactly analogous

to the derivative price curves already described and shown in Fig. 50[1].

If England is supplying cloth to Germany, what she must be doing is to send over all cloth which cannot be disposed of so profitably in England, so that she first satisfies her own market and then supplies her neighbour. And on the other hand, Germany, who wants cloth, first takes up all the cloth her own manu-

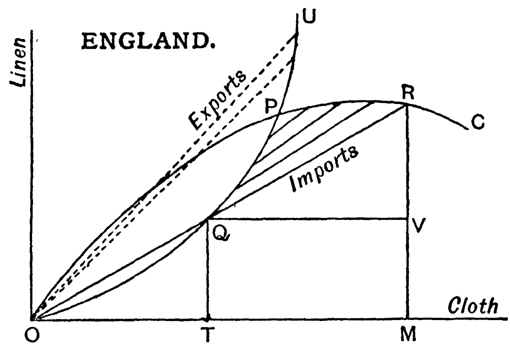


Fig. 56.

facturers can make and then offers her surplus linen to other powers who can supply her with the cloth she requires.

Therefore, in constructing such a curve of export of English cloth, what you have to do is to take the English internal curves of relative utility and cost of production of cloth and linen in Fig. 56 (which is Marshall's form of Fig. 48), and by means of them construct another resultant curve, Fig. 57, with ordinates corresponding in any point, one to Q V in Fig. 56, and the other to R V. In this case if the curve O P U, Fig. 56,

[1] See the Preface.

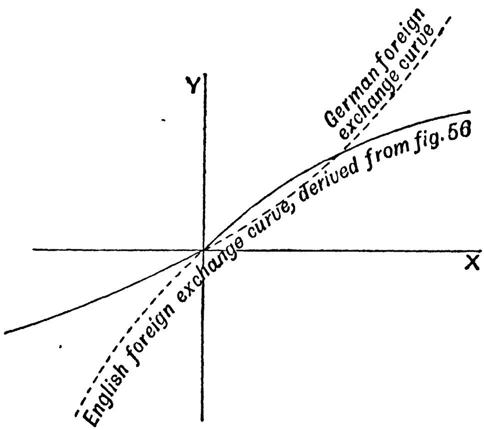
represents the relative utility of linen and cloth in England, and O P C their relative cost of production, P, the point of intersection, would, if there were no foreign trade, fix the relative price and quantities made and sold of cloth and linen in England.

But let us suppose that by the opening up of a trade with Germany an extension takes place in cloth manufacture in England, and England is able to offer cloth to Germany at prices less than those previously prevalent. In that case more cloth will be given for less linen, and the price of linen will rise (say) to R O M, so that a proportion R M of linen is exchanged for M O of cloth, and the price of cloth in terms of linen becomes tan R O M.

In this case the manufacture of English cloth would increase say to O M, of which T M would be exported to Germany to pay for R V of German linen, which they might be very happy to send us at the increased price in cloth which we could afford to give for it. Thus the total amount which the purchasers of linen in England have to offer is O M of cloth, of which O T would go to pay for Q T linen made in England (which at the price tan R O M is all that would be made in England) and Q V would go to Germany to pay for R V of German linen which at the price tan R Q V they would be able to send us. Here we have England exporting cloth to Germany and receiving linen.

Or on the other hand, if, by reason of failure in German linen manufacture, the Germans fell behind in the output of linen, and could only produce it at less cost than we could, so that the price of linen in Germany rose, then the reverse would take place, linen might

now be exported from England and paid for in German cloth. The same effect would be produced if the German manufacturers instead of failing in linen-making, made a brilliant success in cloth-making. That very fact would, even though the linen manufacturers had not really fallen behind, still make the



In each case exports of cloth and imports of linen, whether from or to England or Germany, are shown by a continuous line, and correspond to the continuous lines in Fig. 56. Imports of cloth and exports of linen are shown by a dotted line, and correspond to the dotted lines on Fig. 56.

FIG. 57.

Germans anxious to get rid of the cloth they could so easily make, and thus (greatly to our consumer's profit) they would even dump their cloth on our shores, and thereby give our linen trade an impetus to try and meet it, of course inflicting a corresponding loss on our cloth manufacturers.

If then, by taking the amounts R V of linen imported, and various corresponding amounts Q T of cloth exported from England, you make the continuous curve

shown between X and Y in Fig. 57, you will have a curve representing English exports of cloth as against imports of German linen.

This curve is neither a curve of relative values in use nor of relative costs of production; it is a new sort of curve altogether, a curve compounded of each. It is a true 'demand of linen and supply of cloth curve' for England, depending *both* on the home consumption and production values. It is a curve of over-production of cloth and under-supply of linen. It is shown on Fig. 57, and is a Marshall's foreign exchange curve; and it has two branches, one a continuous line representing a series of amounts by which the demand of England for linen at any given price exceeds her power of supply at that price, and thus indicates a desire to export cloth and import linen; the other, a dotted branch representing a series of amounts by which her supply of linen at a price would exceed her demand, and thus indicates a desire to import cloth and export linen.

And the German foreign exchange curve would likewise have two branches. The dotted branch would represent German import of cloth and export of linen, the continuous would represent export of cloth from Germany and import of linen into Germany. The intersection of these branches determines the exchange. It is even conceivable that these branches might intersect on both sides of the origin, so that there would be two points of stable equilibrium, at one of which England would export cloth to Germany, at the other of which Germany would export cloth to England.

These curves are therefore a pair of over and under production curves exactly analogous to the pair of price curves shown in Fig. 50. They are not subject

to the same laws as the simple curves of internal production and consumption, but they obey laws which have been elaborated with great sagacity and skill by Professor Marshall in the privately printed essay above referred to.

These laws are really derived from a consideration of the effects which the laws of the curves of internal demand and supply have upon the ordinates and abscissae of the international exchange curves. As has been previously pointed out, when the internal demand and supply curves are very steep the growth of the abscissa will be small as compared with the change in the ordinate. This circumstance appears to be the foundation of Professor Marshall's rules for the classes designated by him I. and II.

The difficulties of framing these rules are considerable, because it must be remembered that goods are no longer balanced against money but against one another. And, moreover, linen, instead of being a single commodity the over-supply of which would soon glut the market as far as linen was concerned, is taken as a representative of the whole range of imports. Consequently, the law of diminishing returns is only of partial and questionable application, and the laws hardly obey any laws that are universal.

Moreover, it is clear that completely to determine the exchange between two countries when the conditions of manufacture and of consumption in each are different, you want, not two curves but four, as shown in the diagrams already given. You want two curves showing the relative cost of supply in each country, and also two more showing the relative utility in each country. Given these, you can, as I have endeavoured

to prove, speedily determine which article is going to be exported and which imported to each of the countries, and the amount of each which will be exchanged. If preferred, Marshall's curves may be employed to represent these four curves; and put in the form of a diagram, Fig. 58, similar in character to that which I have before indicated for use with price curves, Fig. 50, and in which

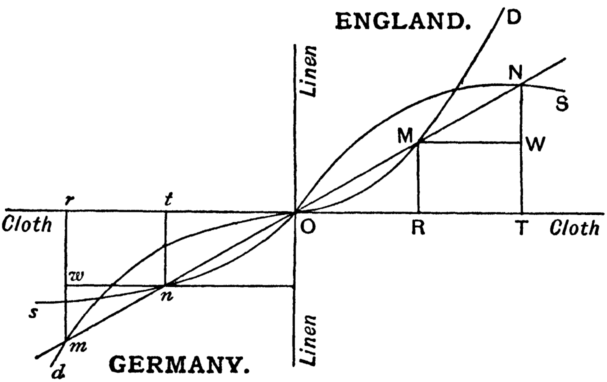


FIG. 58.

a line m O N was drawn such that $m\ n =$ M N, this would show R T cloth being exported from England at a price $=$ tan M O R in linen, in exchange for $w\ m$ of linen at a price $=$ cot M O R in cloth. There you would have the combined action of a pair of value-in-use curves, met by the combined action of a pair of cost-of-production curves, and the ratios of exchange would be shown with perfect symmetry.

But the international exchange curves that seem the most simple are those which deal only with one article and balance it against money, as shown in Fig. 42, *et seq.* They are not unsymmetrical, only their symmetry

is with regard to the axis of Y, the money standard. They seem to me to be more nearly akin to our usual ways of regarding the problem, and to relate more easily to the tables of international values and import duties which are in common circulation, and which are all expressed in money. Added to this, the laws they obey can be made more stringent than the laws of barter curves, so as greatly to simplify the mental consideration of the problem.

Of course, if the amount of one article exported or imported were so enormous as materially to affect the residual amount of money or general values transferred from one to another, then we might have to consider the money standard, against which the article was measured, as no longer stable.

This might, for instance, happen in the case of the importation of American wheat into England; but in most other cases the total imports of one commodity are small, and not sufficient, however they change, to affect the general money standard. Therefore although the price curves previously described, and Marshall's curves, each have their own peculiar merits, for the reasons above given, I am inclined to prefer the system I have advocated for the explanation of the pure theory of international values.

CHAPTER XII

INTEGRAL CURVES

IT is usual in most treatises on curves to assume that the areas between the demand and supply curves represent consumer's surplus value and rent. For reasons previously given I do not think this simple view is permissible, except in a limited number of cases. In all cases, however, we might draw a curve such that

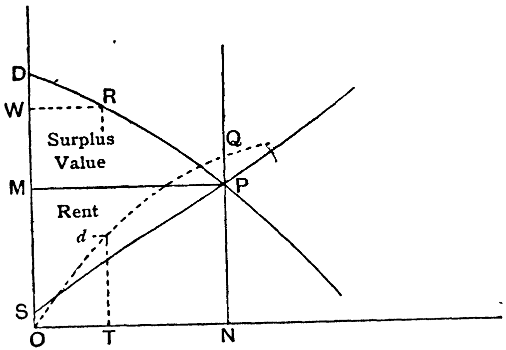


FIG. 59.

its ordinates should represent the consumer's surplus value due to the consumption of an amount of the commodity represented by the abscissa, and thus produce a consumer's surplus value curve, O *d* Q, such that the ordinate T *d* at any point would represent the total consumer's surplus value R D M, obtained by the consumption of an amount represented by the abscissa O T. Similarly we might draw a curve of producer's rent.

These curves would of course be different from the demand and supply curves, and in fact follow a law of their own.

In the case, however, in which we may assume that the consumer's surplus value and the producer's rent are identical with the areas mapped out by the supply and demand curves, these surplus value curves follow a regular law.

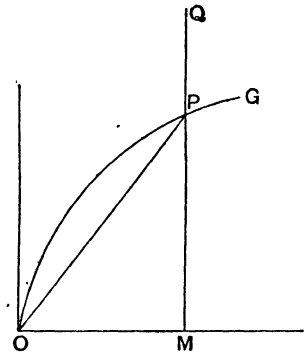


FIG. 60.

Let us consider them as annexed to Marshall's demand and supply curves. For instance, let O G, Fig. 60, be Germany's demand-of-cloth and supply-of-linen curve. Then, if O M of the cloth is supplied the price offered will be tan P O M, and the total linen paid for it will = P M. But the enjoyment in terms of linen of the purchaser will be greater than P M; it will be M Q, where Q is such a point that M Q : M P : : total enjoyment in terms of linen obtained by the consumer of an amount O M of cloth: total amount of linen paid by him. Translating this Marshall's curve into a price curve, we shall have Q M : P M (Fig. 60) : : area R P M O :

area NPMO (Fig. 61). We may construct a curve of the points Q, Fig. 60, corresponding to different values of OM.

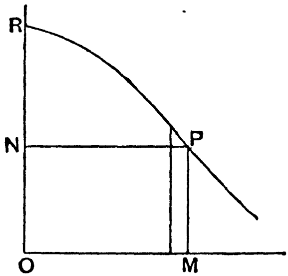


FIG. 61.

Let this curve be shown in the dotted line OQ, Fig. 62. Now the demand price of the final increment of an article is the ratio of the increase of consumer's value

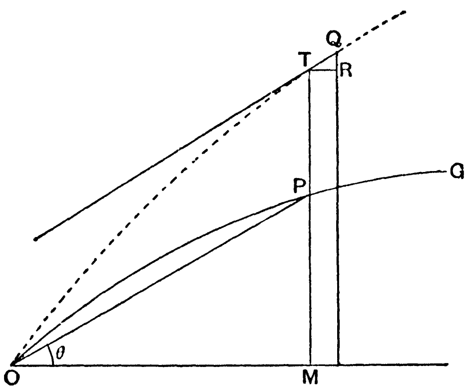


FIG. 62.

which each further increment of quantity produces to the increment of quantity; that is to say, if a small element of the curve at Q be considered, Fig. 62, the

ratio of QR (the increase of consumer's value) to TR (the increase of quantity) is equal to the price. But the ratio of QR to TR is the tangent of the angle QTR, and the price is the tangent of POM; so that it follows that the nature of the curve OQ, shown by the dotted line, must be such that a tangent at Q is always parallel to OP. In other words, as the point P on Marshall's curve sweeps along, the curve OQ of consumer's surplus values must be such as would be followed by a man with a wheelbarrow who kept the position of the barrow always on the ordinate MP, but kept wheeling it along so that its direction of motion was always parallel to OP. On this principle a machine could without great difficulty be designed to describe it. In such a curve, at any point, the abscissa would represent the quantity demanded, the ordinate the total utility to the consumer of the quantity supplied, the tangent to the curve at the point would represent the price per article. The quantity offered in exchange for the quantity demanded would be $X \times$ price per article $= X \tan \theta$.

These consumer's surplus value curves have been greatly developed by Messrs Auspitz and Lieben, and appear to be their favourite mode of expression. For the reasons given above, that they do not really represent either the true rent or the true consumer's surplus utility, they do not appear to me to be likely to prove of wide application as a means of assisting thought upon economic questions.

CHAPTER XIII

ALGEBRAIC TREATMENT OF POLITICAL ECONOMY

In the foregoing pages only geometrical methods have been described. These the author submits are much the most satisfactory for use in economic problems.

It may, however, not be out of place to allude to methods in which only Algebra is employed.

When two quantities are so related that variations in one of them are systematically accompanied by variations in the other, then they are said to be 'functions' one of another. This is a mere expression for their interdependence. Thus if y and x are thus related, y is said to be a function of x, and this is written

$$y = f(x).$$

Thus the use of $f(x)$ is indicative of the fact that a law of some sort binds x to y. It might be an arithmetical law, such as that y was always twice as big as x. In this case we should express it by saying

$$y = 2x.$$

Or again, y might be always equal to the square of x. In this case we should have

$$y = x^2.$$

And of course if y be a price, and x the amount of an article that a market would absorb at that price, then we might have

$$y = f(x).$$

This method of expression, however, involves us in saying that y and x are united by some law. No doubt every series of changes is the result of a law; but most laws in the economic world are so complicated and their elements are so unknown as for us to be no laws at all. There is no 'law' governing the relation of the amount of sugar consumed to the price; that is, no 'law' that can be ever approximately formulated. All that is certain is, that as the price rises the consumption will, if no other causes intervene, fall off; but what the law of the fall would be we can no more say than we can predict the curve that the price of Consols is going to take, or that the index of the barometer is going to follow. The truth is, that these so-called 'laws' are little more than grouped statements of facts, or of suppositions that from experience we may take as approximately representing the facts.

From this point of view, therefore, to express an experimental supply curve as, for instance, that of the price of getting coal, Fig. 35, or of producing a book, Fig. 27, or still more the curves of demand for corn or sugar, by such an expression as

$$y = f(x)$$

is to invest these curves with an apparently simple law-determined character that they do not really possess.

A curve, which can be drawn so as to follow no known law, and yet to have general characteristics, more nearly pictures the problem.

But algebraic expressions for the curves are useful, with due limitations.

They are not limited, as space is, by their dimensions.

For even solid geometry can at most deal with three variants, expressed respectively by length, breadth, and

height. When we get beyond this we want a fourth dimension, and our imagery fails us.

If our object were really to solve economic theorems by Mathematics, or if curves of demand could be shown to follow laws which were capable of expression in algebraic formulae of manageable dimensions, then no doubt Algebra might be used; but the chief function of Mathematics as applied to Economics is, not to solve problems, but to help us to comprehend truths, which when we have comprehended we may discard the Mathematics, as we take down a scaffolding when the building is finished; and the two-dimensional power of plane geometry is in general sufficient for these comparatively modest ends.

Since price depends on marginal cost of production and marginal utility, it is obvious that what is interesting is the rate of relative change of the quantities just about the points at which their value is fixed; that is to say, what change will be involved in the size of y by a very small addition to x?

This consideration involves a branch of Mathematics first conceived by the Greeks in their efforts to find the area of curves, and to solve other similar problems. It was developed in modern times by a succession of mathematicians, from Cavalieri, the pupil of Galileo, to Newton and Leibnitz. A small addition to x is written dx, and a small addition to y is written dy, and the ratio of the changes is therefore $\frac{dy}{dx}$.

When we wish to say that y varies in arithmetical proportion to x, we should write $\frac{dy}{dx} = \text{constant}$.

By making suppositions as to the character of curves,

we can predict by Algebra all the laws abovementioned which affect price. Thus, when we say that the demand curve is always a falling curve, this is as much as to say that when x increases y must always decrease; and hence $\frac{dy}{dx}$ will always be a negative quantity. To say that the demand curve did not cut a rectangular hyperbola, would be to say that $(x+dx)\,y$ must always be greater than $x\,(y-dy)$; meaning that a greater total sum of money is paid for more of an article than for less of it.

It is no part of my purpose to pursue the algebraic part of the subject. It has been excellently treated by Cournot, and in more recent times by Edgeworth and Walras. The only object of this book has been to give the outlines of the geometric methods, which those acquainted with Algebra will have no difficulty in translating into symbolic language.

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